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2-Unweighted aggregate price index for ratios

The average relatives index is a simple, unweighted index number.

It is based on a different idea from the simple aggregative index:

Instead of adding prices directly, we first calculate the relative (ratio) for each commodity separately, then we take the average of these relatives

$$I = \frac{\left(\sum p_1 / \sum p_0\right)}{n} * 100$$

EX4:

Commodity	Price 2020	Price 2024
Rice	1.0	1.8
Oil	2.0	3.5
Sugar	1.5	2.5
Tea	1.0	1.7

$R_r = {}^{1.8}/_{1.0} * 100 = 180$

$R_o = {}^{3.5}/_{2.0} * 100 = 175$

$R_s = 166.7$

$R_t = 170$

$I_p = 172.9\%$

Interpretation:

The general price level increased by 72.9%.

Problem of extreme values (statistically):

If one commodity increases by 500%: $R_1 = 500$

It distorts the average and causes statistical bias.

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Third: Weighted Aggregative Price Index Numbers

In simple index numbers, we implicitly assume that all commodities are equally important, which is not realistic economically.

Example:

• Is an increase in the price of a needle the same as an increase in the price of oil?

Therefore, we need to:

- Give an economic weight to each commodity that reflects its real importance.

1-Weighted price index using Laspeyres' formula

Idea:

- Quantities of the base year are fixed.
- We compare the cost of the same basket in two periods.
- It shows the effect of price changes only

$$I_L = \left({}^{c_1}/_{c_0}\right) * 100$$

$$c_0 = \sum p_0 \, q_0$$

$$c_1 = \sum p_1 \, q_0$$

$$I_L = \left({}^{c_1}/_{c_0}\right) * 100 = \left({}^{\sum p_1 \, q_0}/_{\sum p_0 \, q_0}\right) * 100$$

EX5:

Commodity	(P_0)	(Q_0)	(P_1)
Rice	2	100	3
Sugar	1	80	1.5

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$$\sum p_1\, q_0 = 3 * 100 + 1.5 * 80 = 420$$

$$\sum p_0\, q_0 = 2 * 100 + 1 * 80 = 280$$

$$I_L = \left({}^{420}/_{280}\right) * 100 = 150\%$$

Inflation = 50%

Industrial Example: Pharmaceutical Industry

We want to measure the change in prices of pharmaceutical production inputs between two years.

A pharmaceutical factory uses three main raw materials:

EX6:

Raw Material	Price 2020 (P_0)	Price 2024 (P_1)	Quantity 2020 (Q_0)	Quantity 2024 (Q_1)
Active ingredient (API)	50	80	200	180
Glass containers	10	18	500	600
Chemical reagents	30	45	300	280

$$\sum p_0\, q_0 = \mathbf{50(200) + 10(500) + 30(300) = 10000 + 5000 + 9000 = 24000}$$

$$\sum p_1\, q_0 = \mathbf{80(200) + 18(500) + 45(300) = 16000 + 9000 + 13500 = 38500}$$

$$I_L = \left({}^{\mathbf{38500}}/_{\mathbf{24000}}\right) * 100 = 160.4\%$$

2-Weighted price index using Bash formula

Idea:

- It uses quantities of the current year.
- It usually shows lower inflation.

$L_P = \left({}^{c_1}/_{c_0}\right) * 100$

Where:

$c_0 = \sum p_0\, q_1$

$c_1 = \sum p_1\, q_1$

So

$I_P = \left({}^{c_1}/_{c_0}\right) * 100 = \left({}^{\sum p_1\, q_1}/_{\sum p_0\, q_1}\right) * 100$

EX7:**Agricultural Crops Example**

We measure the change in prices of three crops between:

- **2020: Base year**
- **2024: Current year**

Crop	Price 2020 (P_0) (thousand dinars/ton)	Price 2024 (P_1)	Quantity 2024 (Q_1) (tons)
Wheat	300	450	120
Barley	200	260	150
Tomatoes	250	400	90

$$\sum p_1\, q_1 = \mathbf{450(120) + 260(150) + 400(90)} = 129000$$

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$$\sum p_0\, q_1 = \mathbf{300}(\mathbf{120}) + \mathbf{200}(\mathbf{150}) + \mathbf{250}(\mathbf{90}) = 88500$$

$$I_P = \left({}^{129000}\!/_{88500}\right) * 100 = 145.76\%$$

Agricultural Interpretation

This means that the price level of crops (weighted by 2024 quantities) increased by about:

145.8−100=45.8%145.8 - 100 = 45.8\%145.8−100=45.8%

In other words, the cost of the 2024 production basket would be lower if priced at 2020 prices, but at 2024 prices, it increased by about **45.8%**.

3-**Weighted price index using Marshall–Edgeworth**

The **Marshall–Edgeworth index** is a **weighted index number** that uses the **average of the quantities from the base and current periods** as weights.

It aims to reduce the bias of:

- **Laspeyres index** (uses base-period quantities → tends to overestimate),
- **Paasche index** (uses current-period quantities → tends to underestimate).

Therefore, it is considered a **compromise (intermediate) index** between the two.

$$I_P = \left({}^{c_1}\!/_{c_0}\right) * 100 = \left({}^{\sum p_1\,(q_1+q_0)}\!/_{\sum p_0\,(q_1+q_0)}\right) * 100$$

Weighted used $(q_1 + q_0)$

EX

Commodity	(p_0)	(q_0)	(p_1)	(q_1)
Rice	2	100	3	80
Sugar	1	50	1.5	70

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Step 1: Compute the Marshall–Edgeworth weights $(q_0 + q_1)$

- Rice: $q_0 + q_1 = 100 + 80 = 180$
- Sugar: $q_0 + q_1 = 50 + 70 = 120$

Step 2: Compute the numerator $\sum p_1(q_0 + q_1)$

$$= 3(180) + 1.5(120) = 540 + 180 = 720$$

Step 3: Compute the denominator $\sum p_0(q_0 + q_1)$

$$= 2(180) + 1(120) = 360 + 120 = 480$$

Step 4: Marshall–Edgeworth Price Index

$$I_M = \left({}^{c_1}/_{c_0}\right) * 100 = \left({}^{\sum p_1 (q_1 + q_0)}/_{\sum p_0 (q_1 + q_0)}\right) * 100 = \frac{720}{480} * 100 = 150$$

Result and Interpretation

Marshall–Edgeworth Price Index = 150
This means the overall price level (for rice and sugar) **increased by 50%** from the base period to the current period.

industrial Question (Index Numbers)

A manufacturing company uses two main raw materials in its production process: **Steel** and **Plastic**.
The prices and quantities in the base period and the current period are given below:

Material	(p_0)	(q_0)	(p_1)	(q_1)
Steel	500	40	650	30
Plastic	200	60	260	80

Required:

1. Calculate the **Marshall–Edgeworth Weighted Price Index**.
2. Interpret the result from an **industrial cost perspective**.

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Solution

Step 1: Compute the weights $(q_0 + q_1)$

- Steel: $40 + 30 = 70$
- Plastic: $60 + 80 = 140$

Step 2: Compute the numerator

$$\sum p_1(q_0 + q_1)$$

$$= 650(70) + 260(140) = 45{,}500 + 36{,}400 = 81{,}900$$

Step 3: Compute the denominator

$$\sum p_0(q_0 + q_1)$$

$$= 500(70) + 200(140) = 35{,}000 + 28{,}000 = 63{,}000$$

Step 4: Marshall–Edgeworth Price Index

$$P_{ME} = \frac{81{,}900}{63{,}000} \times 100 = 130$$

Interpretation (Industrial Context)

The **Marshall–Edgeworth price index equals 130**, which means that the **cost of raw materials has increased by 30%** compared with the base period.

Industrial implication:

- Production costs have risen significantly.
- If selling prices are unchanged, **profit margins will decrease**.
- Management may need to:
  - Adjust product prices,
  - Improve production efficiency,
  - Renegotiate supplier contracts.

Short exam-ready conclusion

The Marshall–Edgeworth index shows a 30% increase in raw material prices, indicating a substantial rise in industrial production costs.

4-**Weighted price index using Fisher formula**

To balance the bias of the Laspeyres and Paasche indices:

$$I_F = \sqrt{I_L I_P}$$

Why is it called "ideal"?

- Because it balances the bias of Laspeyres and Paasche

EX8: (Household Goods)

Commodity	Price 2020 (P_0)	Quantity 2020 (Q_0)	Price 2024 (P_1)	Quantity 2024 (Q_1)
Rice (kg)	1,500	120	2,500	110
Oil (liter)	2,000	80	3,800	70
Sugar (kg)	1,000	150	1,900	140
Gas (cylinder)	12,000	20	18,000	18

$$\sum p_0\, q_0 = \mathbf{180000 + 160000 + 150000 + 240000 = 730000}$$

$$\sum p_1\, q_0 = \mathbf{300000 + 304000 + 285000 + 360000 = 1,249,000}$$

$$I_L = \left(\mathbf{1,249,000}/\mathbf{730000}\right) * 100 = 171.10\%$$

$$\sum p_1\, q_1 = \mathbf{275000 + 266000 + 266000 + 324000 = 1,131,000}$$

$$\sum p_0\, q_1 = \mathbf{165000 + 140000 + 140000 + 216000 = 661,000}$$

$$I_P = \left(\mathbf{1,131,000}/\mathbf{661,000}\right) * 100 = 171.10\%$$

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$$I_F = \sqrt{I_L I_P} = \sqrt{\mathbf{1,131,000} * 171.10} = 171.10\%$$

EX9:

Commodity	Price 2020 (P_0)	Quantity 2020 (Q_0)	Price 2024 (P_1)	Quantity 2024 (Q_1)
Rice (kg)	1,500	120	2,700	90
Chicken (kg)	4,500	60	7,500	40
Tomatoes (kg)	800	100	1,600	140
Petrol (liter)	450	300	750	220

$$\sum p_0\, q_0 = \mathbf{180,000 + 270,000 + 80,000 + 135,000 = 665,000}$$

$$\sum p_1\, q_0 = \mathbf{324,000 + 450,000 + 160,000 + 225,000 = 1,159,000}$$

$$I_L = \left(\mathbf{1,159,000}/\mathbf{665,000}\right) * 100 = 174.29\%$$

$$\sum p_1\, q_1 = \mathbf{243,000 + 300,000 + 224,000 + 165,000 = 932,000}$$

$$\sum p_0\, q_1 = \mathbf{135,000 + 180,000 + 112,000 + 99,000 = 526,000}$$

$$I_P = \left(\mathbf{932,000}/\mathbf{526,000}\right) * 100 = 177.19\%$$

$$I_F = \sqrt{I_L I_P} = \sqrt{174.29 * 177.19} = 175.7\%$$

Economic Interpretation (Iraq)

175.7−100=75.7%

This means that the price level of the selected Iraqi goods basket increased by about **75.7%** between 2020 and 2024.

Why is there a difference between the indices?

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Because quantities changed clearly in 2024:

- Consumption of chicken, rice, and petrol decreased (adaptation to higher prices).
- Quantity of tomatoes increased (cheaper or more available substitute).

So:

- **Laspeyres** uses old quantities (Q_0).
- **Paasche** uses new quantities (Q_1).
- **Fisher** takes the geometric average of both (balanced estimate).
- **Marshall** index between the Laspeyres and Paasche It is considered a compromise (intermediate)

Example A: Building an "Iraqi Cost of Living Basket" using Laspeyres Index

Base period = Month (0)

Comparison period = Month (1)

Items represent spending of a working family.

EX10:

Item	P_0 (IQD)	Q_0	P_1 (IQD)	$P_0 \times Q_0$	$P_1 \times Q_0$
Rice (kg)	2,000	20	2,600	40,000	52,000
Chicken (kg)	6,000	10	7,200	60,000	72,000
Petrol (liter)	750	120	900	90,000	108,000
Monthly rent	400,000	1	480,000	400,000	480,000
Monthly telecom	25,000	1	30,000	25,000	30,000

$$\sum p_1 q_0 = 742000\ IQD$$

$$\sum p_0 q_0 = 615000\ IQD$$

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$L = 120.65\%$

Laspeyres basket index (L) = 120.65 ⇒ the cost increased by about 20.65%.

Practical interpretation:

An organization can estimate internal inflation for employees' needs (cost of living) or update an operating budget.

EX11: "Project Cost Index" using the Laspeyres method

Inputs: cement, steel, wages, transport fuel.

The quantity Q_0 represents the original project plan

Input	P_0 (IQD)	Q_0	P_1 (IQD)	$P_0 \times Q_0$	$P_1 \times Q_0$
Cement (ton)	120,000	50	150,000	6,000,000	7,500,000
Steel (ton)	900,000	20	1,100,000	18,000,000	22,000,000
Wages (day)	35,000	300	45,000	10,500,000	13,500,000
Transport fuel (liter)	800	2000	950	1,600,000	1,900,000

$$\sum p_1 \, q_0 = 44{,}900{,}000 \; IQD$$

$$\sum p_0 \, q_0 = 36{,}100{,}000 \; IQD$$

$L = 124.38\%$

Project cost index = 124.38 ⇒ the project cost increased by about 24.38%.

Interpretation for Industrial Applications

- **Laspeyres Index**
  Useful for **quick cost estimation**, but may exaggerate cost increases because it assumes unchanged production quantities.

- **Paasche Index**
  Reflects **current production patterns**, but may underestimate cost increases due to substitution toward cheaper inputs.
- **Marshall–Edgeworth Index**
  Provides a **balanced and more realistic measure** of input cost changes in industrial settings, especially when production quantities change over time.

تفسير التطبيقات الصناعية

مؤشر لاسبير: مفيد لتقدير التكاليف بسرعة، ولكنه قد يبالغ في تقدير الزيادات في التكاليف لأنه يفترض ثبات كميات الإنتاج.

مؤشر باش: يعكس أنماط الإنتاج الحالية، ولكنه قد يقلل من تقدير الزيادات في التكاليف نتيجة استبدال المدخلات بمدخلات أرخص.

مؤشر مارشال-إيدجوورث: يوفر مقياسًا متوازنًا وأكثر واقعية لتغيرات تكلفة المدخلات في البيئات الصناعية، خاصةً عند تغير كميات الإنتاج بمرور الوقت.

Aspect	**Laspeyres Index**	**Paasche Index**	**Marshall–Edgeworth Index**
Type of index	Weighted price index	Weighted price index	Weighted price index
Weights used	Base-period quantities (q_0)	Current-period quantities (q_1)	Average quantities ((q_0 + q_1))
Formula	$I_L = \frac{\sum p_1\, q_0}{\sum p_0\, q_0} * 100$	$I_P = \frac{\sum p_1\, q_1}{\sum p_0\, q_1} * 100$	$I_M = \frac{\sum p_1\,(q_1 + q_0)}{\sum p_0\,(q_1 + q_0)} * 100$