$$A = \left\{ (x,y) : 0 \leq x \leq 2 : 0 \leq y \leq \frac{1}{2} \right\},$$

$$B = \{ (x,y) : 0 \leq x \leq 1 ; 0 \leq y \leq 1 \},$$

$$C = \{ (x,y) : 0 \leq x \leq y \leq 1 \},$$

$$D = \left\{ (x,y) : 0 \leq x \leq 1 ; 0 \leq y \leq \frac{1}{2} \right\}.$$

Then the following set relations follow:
$D \subset A$, $D \subset B$; $A \cap B = D$, $B \cup C = C \cup D$;

$$A' = \left\{ (x,y) : 0 \leq x \leq 2;\ \frac{1}{2} < y \leq 1 \right\}$$

and

$$A/D = \left\{ (x,y) : 1 < x \leq 2 ; 0 \leq y \leq \frac{1}{2} \right\}$$

The set operations can be represented by diagrams as in Fig 1.1 which are called Venn-diagrams

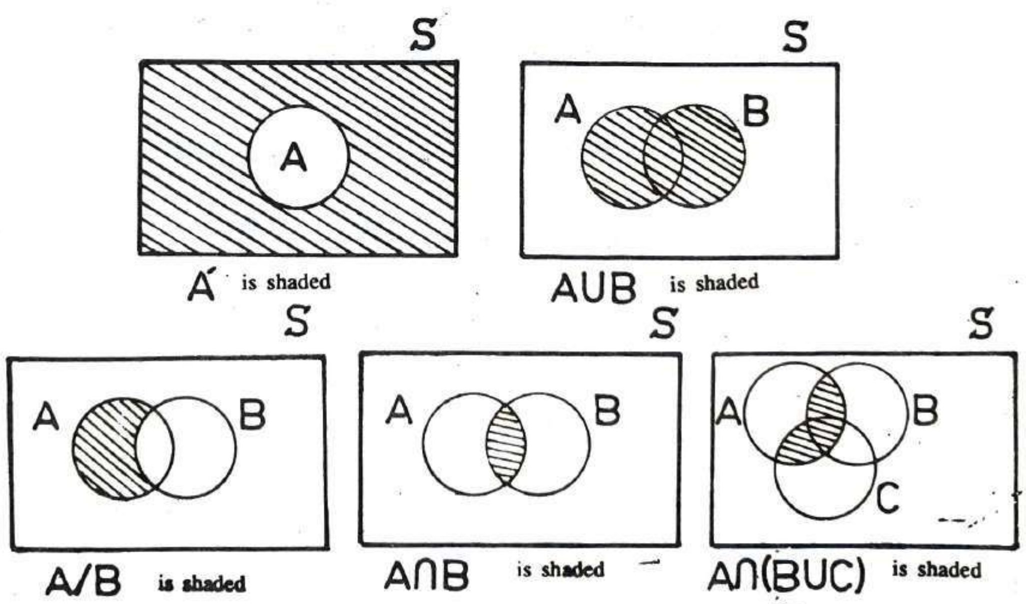

Fig. 1.1

Definition 1.1.6

Let A and B be two sets. The *product set* of A and B, denoted by AxB, is the set of all ordered pairs (x,y) where $x \in A$ and $y \in B$, i.e.

$$AxB = \{ (x,y) : x \in A \text{ and } y \in B\} \quad .... (1.1.9)$$

The product of a set A with itself is denoted by A^2. Equation (1.1.9) can be extended to any finite number of sets $A_1, A_2, ..., A_n$ as

$$A_1 \times A_2 \times .... \times A_n = \{ (x_1, x_2, ..., x_n) : x_i \in A_i \text{ for } i=1,2, ... n\}$$

For example :

Let A= {1,2,3} and B= {4,5}. Then

AxB = { (1,4), (1,5), (2,4), (2,5), (3,4), (3,5)}.

Definition 1.1.7

The set A is called *finite* if it is empty or if it consists of exactly n elements where n is a positive integer. Otherwise it is called an *infinite* set.

An infinite set is said to be *countable* if its elements can be put in one-to-one correspondence with the natural numbers 1,2,3,4, ... In this case it is called *countably infinite*. If the set has as many points as there are in some interval of real numbers [a,b], it is called a *noncountably infinite* set.

Example 1.1.5

1. Let X be the set of the days of the week, i.e.
   X = { Sun., Mon., Tue., Wed., Thur., Fri., Sat.}, then X is finite
2. Let Y be the set of all negative odd integers, i.e.
   Y = { ....., -7, -5, -3, -1}, then Y is countably infinite.
3. Let I be the unit interval of real numbers, i.e.
   I = {x: $0 \leq x \leq 1$}, then I is non countably infinite.
4. Let S = {x: x is a 2nd year student of statistics in our college}.

Then S is finite.

We shall use the word class or family of sets for the set of sets, like

C = { {2}, {1,2}, {5,6,7,8}}

Note that the members of C are also sets.

Definition 1.1.8

The class of all subsets of any set A is called the *power set* of A and denoted by $\mathcal{P}(A)$. If A is finite and has n elments, then $\mathcal{P}(A)$ will have 2^n elements.

For example : Let A = {1,2,3}, then
$\mathcal{P}$ (A) = { ϕ , {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, A}

A *partition* of a set A is a subdivision of A into nonempty subsets which are disjoint and whose union is A. The subsets in a partition are called *cells*.

Example 1.1.6

Consider the following classes of $A = \{0,1,2,3,4,5,6,7\}$

i. $C_1 = \{ \{0,1,4\}, \{2,3\}, \{5,6\} \}$

ii. $C_2 = \{ \{0,1,2\}, \{3,4,7\}, \{5,6\} \}$

iii. $C_3 = \{ \{0,1,2\}, \{2,3,4\}, \{5,6,7\} \}$

C_1 is not a partition of A, since $7 \in A$ but 7 does not belong to any cells, C_2 is a partition of A, and C_3 is not a partition of A since 2 belongs to both $\{0,1,2\}$ and $\{2,3,4\}$

1.2- Some Fundamental Theorems

The set operations of complement, union and intersection have been defined in Sec. 1.1. These operations satisfy a number of laws, some of which follow, stated as theorems. The proof of some of them is omitted.

Let A,B and C be any subsets of the space S .

1. Idempotent laws (Reflexive)
   $A \cup A = A$ and $A \cap A = A$ (1.2.1)

2. Associative laws:
   $A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C$
   $A \cap (B \cap C) = (A \cap B) \cap C = A \cap B \cap C.$ (1.2.2)

3. Commutative laws:
   $A \cup B = B \cup A$ and $A \cap B = B \cap A$ (1.2.3)

4. Distributive laws:
   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (1.2.4)
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (1.2.5)

To prove (1.2.4), let

$x \in A \cup (B \cap C) \Longleftrightarrow x \in A$ or $x \in B \cap C$ by (1.1.4)

$\Longleftrightarrow x \in A$ or ($x \in B$ and $x \in C$) by (1.1.7)

$\Longleftrightarrow$ ($x \in A$ or $x \in B$) and ($x \in A$ or $x \in C$)

$$\Longleftrightarrow (x \in A \cup B) \text{ and } x \in (A \cup C)$$
$$\Longleftrightarrow x \in (A \cup B) \cap (A \cup C)$$

Therefore $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
In the same way, we can prove (1.2.5).

5. Identity laws:

$A \cup \phi = A$ and $A \cap \phi = \phi$ (1.2.6)

$A \cup S = S$ and $A \cap S = A$ (1.2.7)

6. Complement laws:

$A \cup A' = S$ and $A \cap A' = \phi$ (1.2.8)

$(A')' = A$, $S' = \phi$, and $\phi' = S$ (1.2.9)

7. De Morgan's laws:

$(A \cup B)' = A' \cap B'$ (1.2.10)

$(A \cap B)' = A' \cup B'$ (1.2.11)

To prove (1.2.10), let

$x \in (A \cup B)' \Longleftrightarrow x \in S$ and $x \notin (A \cup B)$ by (1.1.1)

$\Longleftrightarrow x \in S$ and $(x \notin A$ and $x \notin B)$ by (1.1.5)

$\Longleftrightarrow (x \in S$ and $x \notin A)$ and $(x \in S$ and $x \notin B)$

$\Longleftrightarrow (x \in A')$ and $(x \in B')$

$\Longleftrightarrow x \in A' \cap B'$ by (1.1.7)

Therefore $(A \cup B)' = A' \cap B'$.
To prove (1.2.11) we follow the same procedures.

The above laws can be provided by using the Venn-diagrams. For example, we can prove the distributive law (1.2.5) as in Fig. 1.2.