Formulating the mathematical model

To formulate the Linear Programming (L.P.) Model, the following steps are generally followed:

1-Determine the decision variables (the variable factors), which is usually the item required to be determined in the problem under study and are expressed using .algebraic symbols

2-Define the problem constraints, which are typically the available resources in the .problem under study and are all expressed using linear inequalities or equalities

3-Determine the objective function, the equation that measures the profit .((maximization) or the cost (minimization

4-All variables in the constraints must be represented in the objective function, and .each variable must have a coefficient of 1

The following examples illustrate the steps above:

.**Example (1-1**): A leather workshop produces two types of leather bags, A and B

Producing one bag of type A requires 2 meters of leather and 3 man-hours of labor .weekly

Producing one bag of type B requires 1 meter of leather and 1 man-hour of labor .weekly

The profit from one bag of type A is 300 Dinars, and the profit from one bag of type B is 200 dinars, knowing that the weekly available quantity of leather is 100 meters with 120 weekly working hours. Construct a linear programming (L.P.) model to find the number of bags produced weekly (of both types A and B) and maximize the factory's weekly profit

:**Solution**

Two types of bags, A and B, must be produced to maximize profit. The first step is to determine the decision variables, which represent the number of bags produced of :each type, as follows

:The decision variables are

.x1: The expected number of bags of type A to be produced

.x2: The expected number of bags of type B to be produced

After determining the decision variables, we determine the constraints of the problem. In this case, the constraints are limited by the available resources (leather, .(working hours

.Producing one bag of type A requires 2 meters of leather

Just as x1 represents the number of bags produced of type A, the quantity of leather .required to produce type A bags is 2x1

Similarly, producing type B bags requires x2 of leather. Thus, the total amount of hides required to produce types A and B is: 2X2+X2

which must not exceed the available amount of hides, which is 100 meters. Therefore, the leather constraint is: 2X2+X2≤100

As for time, producing one bag of type A requires 3 weekly working hours. This : means that the time required to produce type A bags is 3X1. Similarly, the time required to produce type B bags is X2. Therefore, the time required to produce both types of bags A and B is 3X1 + 2X2, which must not exceed the weekly working hours .of 120. Therefore, the working hours constraint is: 3X1 + 2X2 ≤ 120

Producing bags of either type takes two probabilities: either zero production or no production if the profit from producing one type is low compared to the other type, and the resources required to produce it are greater than the resources required to produce the other type. This means that producing one type of bag yields a greater profit for the factory than producing both types. Based on this, the production of any type of bag must be greater than or equal to zero. This is called the non-negativity .constraint, i.e., X1, X2 ≥ 0

The final step in developing the model is to determine the goal sought by the decision maker, which in this case represents maximizing daily profit. For a factory, the profit from producing Type A bags is 300 x 1 plus the profit from producing Type .B bags is 200 x 2. Therefore, the total profit from production is 300 x 1 + 200 x 2

Assuming that Z = 300 x 1 + 200 x 2, the goal is to maximize Z. Therefore,

: the objective function is written as follows

Max $Z=300X1+200X2$

:Therefore, the final linear programming (L.P.) problem is as follows

Max $Z=300X1+200X2$

S.T.:

$2X2+X2\leq 100$

$3X1+2X2\leq 120$

$X1,X2 \geq 0$

Example (1-2): A food company produces two types of food products, A and B. The production of both types requires three kinds of raw materials: I, II, and III. The production of one unit of either food type requires the amount of raw materials shown in Table below

Type of Raw Material	A	B
I	2	3
II	1	2
III	3	1

The available amount of raw material type I is 40 kg per day, the available amount of raw material type II is 20 kg per day, and the available amount of raw material type III is 30 kg per day. What are the number of units produced daily of the two food types A and B that will lead to maximizing the company's daily profit rate, given that the profit per unit of type A is 20 Dinars and the profit per unit of type B is 25 Dinars

Solution: Step 1: The first step to form the Linear Programming(LP) model is to define the Decision Variables, which represent the number of units produced daily for types A and B

as follows:

X1: **The number of units expected to be produced of type**

X2: **The number of units expected to be produced of type B**

Step Two: Defining Constraints Raw Material Type I Constraint The unit of type A needs 2 units of Raw Material I, and the unit of type B needs 3 units of Raw .Material I

One unit of type A requires two raw materials of one type. Therefore, the quantity of raw material I required to produce type A is 2X1. Likewise, the quantity of raw material I required to produce type B is 3X2. The total quantity of raw material I required to produce both types A and B is: 2X1 + 3X2. This must not exceed the available raw material, which is 40 kg. Therefore, the entry for raw material I is as :follows

$2X1 + 3X2 \leq 40$

Similarly, one unit of type A requires one raw material of type II. Therefore, the quantity of raw material II required to produce type A is 1X. Likewise, the quantity of raw material II required to produce type B is 2X2. Therefore, the total quantity of raw material II required to produce both types A and B is: X1 + 2X2, which must not exceed the available raw material II (20 kg Therefore, the entry for raw material II is In the following form: $X1 + 2X2 \leq 20$

Using the same method as the previous two constraints, one unit of type A requires) 3 units of type III raw materials.) Therefore, the quantity of raw materials III required to produce type A is 3X1, and the quantity of raw materials III required to produce type B is X2. Therefore, the total quantity of raw materials III required to produce both types A and B is: $30 \geq 3X1 + X2$. This is in addition to the non-negativity .constraints: $X1, X2 \geq 0$

After determining the decision variables and then determining the constraints of the linear programming (L.P.) model, it is necessary to determine the objective function, which represents maximizing the company's daily profit rate resulting from the profit from producing type A, which is X1 = 20, plus the profit from producing type B, which is X2 = 25. Therefore, the objective function is in the following form: Max Z = 20X1 + 25X2

Therefore, the final linear programming (L.P.) problem is in the following form

:

Max Z = $20X_1+25X_2$

:.S.T

$X_1+3X_2 \leq 402$

$X_1+2X_2 \leq 20$

$X_1+X_2 \leq 303$

$X_1,X_2 \geq 0$

Example (3-1): A company produces four types of bicycles (D, C, B, A). The company has two production lines, and to produce any of the four types, it must pass through these two lines. Producing bicycle (A) requires 2 labor hours for the first production line and 3 labor hours for the second production line, while producing bicycle (B) requires 1 labor hour for the first production line and 4 labor hours for the second production line. Producing bicycle type C requires 2 labor hours for the first production line and 2 labor hours for the second production line. Producing bicycle type D requires 3 labor hours for the first production line and 4 labor hours for the second production line. The available weekly labor hours for the two production lines are 150 labor hours for the first production line and 120 labor hours for the second production line. For the second line, the company sells bicycle A for 15,000 dinars, with a production cost of 13,000 dinars. Bicycle B sells for 17,000 dinars, with a production cost of 14.5 thousand dinars. Bicycles D and C sell for 22,000 dinars, with a production cost of 18.5 thousand dinars. In addition, the company incurs the cost of transporting the bicycles from the company to the marketing outlets, which is 1,000 dinars for bicycle A, 500 dinars for bicycle B, 500 dinars for bicycle C, and 1,500 dinars for bicycle D. A linear programming (LP) model is required to determine the weekly production quantity for each type of bicycle, .which maximizes the company's profits

:Solution

The decision variables for the problem represent the number of bicycles produced :weekly for the four types, as follows

x1: Number of bicycles of type A expected to be produced

x2: Number of bicycles of type B expected to be produced

x3: Number of bicycles of type C expected to be produced

x4: Number of bicycles of type D expected to be produced

After determining the decision variables, the problem constraints are determined, :representing the weekly working hours for the two production lines, as follows

Production line 1: $2X1 + X2 + 2X3 + 3X4 \leq 150$

Production line 2: $X1 + 4X2 + 2X3 + 4X4 \leq 120$

The objective function for the problem represents the net profit resulting from the :sale of bicycles for the four types, and is calculated according to the equation

(Net profit = selling price - (production cost + transportation cost

:Therefore, the net profit for the four types of bicycles is

Net profit for bicycle type A equals (15 - (13 + 1)) = 1

Net profit for bicycle type B equals (17 - (14.5 + 0.5)) = 2

Net profit for bicycle type C equals (22 - (18.5 + 0.5)) = 3

Net profit for bicycle type D Equals (22 - (18.5 + 1.5)) = 2

Therefore, the final formula for the objective function is as follows: Max $Z = X1 + 2X2 + 3X3 + 2X4$

And the final formula for the linear programming model is as follows:

Max $Z= X1+2X2+3X3+2X4$

$2X1 +X2 +2X3 +3X4\leq 150$

$3 X1+4X2+2X3 + 4X4\leq 120$

$X1,X2,X3 ,X4 \geq 0$

Example(1- 4) :A dairy company produces two types of cream. The company is required to produce at least 500 units of the first type, 1,000 units of the second type, and 2,000 units of both types daily. The cost of producing one unit of the first type is 95%, and the success rate of marketing the second type is 98%. The cost of success resulting from failure to market the first type is 3 dinars, and the cost resulting from failure to market the second type is 1 dinar per unit. The company wants to develop an optimal production plan for both types of cream to minimize .costs

Solution :

:The decision variables for the problem are as follows

.x1: The expected number of units of the first type of cream produced daily

.x2: The expected number of units of the second type of cream produced daily

:The constraints of the problem are as follows

$X1 + X2 \geq 2000$

$X1 \geq 500$

$X2 \geq 1000$

After determining the decision variables and constraints for the problem, we determine the objective function, which in this problem is to minimize production costs. These costs are of two types: the first represents production costs, and the second represents the costs of failure in marketing. Therefore, the total cost of :production is calculated according to the following equation

(Total Cost = Production Cost + (Cost of failure in marketing * Probability of error

:Therefore, the total production costs for the two types of values are

$15.15 = (3)0.05 + 15$

$20.02 = (1)0.02 + 20$

:The linear programming (LP) model for the problem is as follows

Min Z = 15.5X1 + 20.02 X2

X1+X2≥2000

X1≥500

X2≥1000

X1,X2≥0