

## Game Theory

Introduction: Game theory (also known as game theory) is credited to John von Neumann, who published his research in 1928. It was first applied to economic and military matters in 1944, and has since continued to develop to the present day.

Game theory deals with competition between players, companies, or institutions. The two players can be individuals, institutions, or corporations. The outcome of the game is detrimental to one player and detrimental to the other. There are two types of outcomes in games:

1 - Zero-sum games: In other words, one-quarter of the players lose = one-quarter of the players lose.

2- Non-zero-sum games: In other words, the players lose = one-quarter of the players lose, and vice versa.

A game is defined as the set of rules and laws that govern decision-making in an administrative, economic, or military matter.

### Basic Concepts in Game Theory

**1-Players :** A game can be between two or more teams. A team can be an individual, a company, or an institution.

**2-Pay of Matrix:** It consists of:

A. Payouts: This can be a win or a loss.

B. Strategies: These are the plans developed by the competitors, and each player has their own strategies.

The Outcome of the Game: There are two outcomes for a game:

A. Zero-Sum Game: This means that what the first team wins is exactly equal to what the second team loses.

B. Non-Zero-Sum Game: Here, what the first team wins is not necessarily exactly equal to what the second team loses.

## First: The Zero-Sum Game

Definition: A game in which one player's (countries/parties) gain is equal to the other player's loss. That is, the sum of the gains and losses ultimately equals zero (0).

Objective: Each player plays to achieve their best strategy (equilibrium strategy).

Method: The max min and mini max methods.

Steps described for finding the value and equilibrium strategy:

### 1. Max min - Player A's strategy:

Process: "We find the smallest value in each row and place it in a new column in the game matrix, labeled Min (the minimum of the rows). Then, from this column, we choose the largest value (Max Min)."

Meaning: Player A (whose payoff matrix represents his gains) first chooses the lowest possible gain in each row (in each of his strategies, assuming his opponent will play the best way to his disadvantage). Then, he chooses the largest of these minimum values to guarantee himself the largest minimum gain.

Value: This value is the "value of the game for player A," denoted  $V_1$ .

Goal: Maximize his minimum gain.

### 2. Mini max - Player B's strategy:

Process: "We find the largest value in each column and place it in a new row of the game matrix, calling it Max (maximum columns). Then, we take the smallest value (Min Max) from this row."

Meaning: Player B (whose payoff matrix represents his losses) first chooses the largest possible loss in each column (in each of his opponent's strategies, assuming his opponent will play the best winning strategy). He then chooses the smallest of these maximum values to minimize the maximum loss.

Value: This value is the "value of the game for player B," denoted  $V_2$ .

Objective: Minimize his maximum loss.

### 3. Saddle Point / Equilibrium Strategy:

Condition: "The game sum is zero if the game value is ( $V = V_1 = V_2 = 0$ ) (Note: The condition written in the text appears abbreviated or inaccurate. The correct condition for the saddle point is ( $V_1 = V_2$ )).

Correct meaning: If the first player's minimum maximum value ( $V_1 = \text{Max Mini}$ ) equals the second player's minimum maximum value ( $V_2 = \text{Min Max}$ ),

Example:- If the following payment matrix is:

اللاعبون	$y_1$	$y_2$	$y_3$	$y_4$
$x_1$	8	-2	9	-3
$x_2$	6	5	6	8
$x_3$	-2	4	-9	-1

Required: Find the value of the game for players A and B and the equilibrium point.

Solution: We rewrite the matrix and find the column Min and the column Max.

اللاعبون	$y_1$	$y_2$	$y_3$	$y_4$	Min
$x_1$	8	-2	9	-3	-3
$x_2$	6	5	6	8	5
$x_3$	-2	4	-9	-1	-9
Max	8	5	9	8	Max, Min =5

$$v_1 = \max, \min = 5$$

$$v_2 = \min, \max = 5$$

The playing strategy for player A

استراتيجية اللعب بالنسبة للاعب A

is equal to the playing strategy for player B.

تساوي استراتيجية اللعب بالنسبة B

The equilibrium point is  $v_1 = v_2 = 5$

نقطة التوازن  $v_1 = v_2 = 5$

The value of the game is  $V = v_1 = v_2$

اما قيمة المباراة فأنها  $V = v_1 = v_2$