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Chain Index Numbers

1. Theoretical Introduction

In all previous cases, we used a **fixed base year**.

However, in long time series, a serious problem appears:

The base year becomes old and no longer represents the real economic situation.

The solution is: Chain Index Numbers.

In chain indices, the base year changes in every period.

Instead of always comparing with year 0, we compare:

Each year with the previous year.

That is:

- 2021 compared with 2020
- 2022 compared with 2021
- 2023 compared with 2022
- and so on…

Weighted Chain Index Numbers

1-Laspeyres Chain Index (Weighted Form)

$$I_{Li} = \left(\sum p_i \, q_{t-1} \Big/ \sum p_{i-1} \, q_{t-1} \right) * 100$$

Weighted Industrial Chain Example

Factory production over three years

Year	Quantity	Price
2021	100	10
2022	130	12

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Year	Quantity	Price
2023	160	15

Laspeyres Chain Quantity Index

$$I_{21,22} = \left({}^{130 * 10}/_{100 * 10}\right) * 100 = 130$$

$$I_{22,23} = \left({}^{160 * 12}/_{160 * 12}\right) * 100 = 123$$

المؤشر التراكمي

$$\boldsymbol{I_{Li} = I_{21,22} * I_{22,23} = 160}$$

أي أن الإنتاج الحقيقي زاد **60%**.

Why do we use chain indices? (Advantages)

1. They continuously reflect the current economic reality.
2. They reduce distortion from an outdated base year.
3. They are suitable for long time series.
4. They are the basis of modern national accounts.

Relation to National Accounts

Today, most countries calculate:

Real GDP = Chain Volume Index

instead of:

Fixed Base GDP

because:

- The economic structure changes every year.
- Weights must be updated continuously.

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Advanced Applied Example (GDP)

An economy with three sectors

Year	Agriculture	Industry	Services
2020	100	200	300
2021	120	230	330
2022	150	260	380

نحسب النمو السنوي الحقيقي ثم نربطه تسلسليًا:

$$I_{20,21} = \left({}^{680}/_{600}\right) * 100 = 113.3$$

$$I_{21,22} = \left({}^{790}/_{680}\right) * 100 = 116.2$$

المؤشر التراكمي

$$\boldsymbol{I_{Li} = I_{20,21} * I_{21,22} = 131.7}$$

أي نمو حقيقي تراكمي31.7%

الأرقام القياسية التسلسلية هي مؤشرات تعتمد على مقارنة كل فترة بالفترة السابقة لها، وتُستخدم لتفادي تشوه سنة الأساس في السلاسل الزمنية الطويلة، وتُعد الأساس المعتمد حاليًا في حساب الناتج المحلي الحقيقي والمؤشرات الاقتصادية الحديثة، لما توفره من دقة وواقعية عالية في تمثيل التغير الاقتصادي الديناميكي.

مثال عددي (سلعتان – ثلاث سنوات)

Year	Price A	Quantity A	Price B	Quantity B
2021	10	50	20	30
2022	12	60	25	40
2023	15	70	30	45

Step 1: Laspeyres Index from 2021 to 2022

$$I_{21,22} = \left({}^{50 * 12 + 25 * 30}/_{10 * 50 + 20 * 30}\right) * 100 = 122.7$$

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Step 2: Laspeyres Index from 2022 to 2023

$$I_{22,23} = \left({}^{15*60+30*40}/_{12*60+25*40}\right) * 100 = 122.1$$

Step 3: Cumulative (Chain) Laspeyres Index

$$I_{21,23} = 122.7 * 122.1 = 149.8$$

This means that prices increased by 49.8% over the two periods

2-Paasche Chain Index (Weighted Form)

General formula

$$I_{Pi} = \left({}^{\sum p_i\, q_t}/_{\sum p_{i-1}\, q_t}\right) * 100$$

That is:

- We fix the quantities of the current period.
- We measure price changes from **t−1 to t**.

Paasche from 2021 to 2022

Step 1: Paasche Index from 2021 to 2022

$$I_{21,22} = \left({}^{12*60+25*40}/_{10*60+20*40}\right) * 100 = 122.9$$

Step 2: Paasche Index from 2022 to 2023

$$I_{22,23} = \left({}^{15*70+30*45}/_{12*70+25*45}\right) * 100 = 122.1$$

Step 3: Cumulative (Chain) Paasche Index

$$I_{21,23} = 122.9 * 122.1 = 150$$

This means that prices increased by 49.8% over the two periods

اي الأرقام القياسية التسلسلية، يُحسب مؤشر لاسبيرس بمقارنة أسعار الفترة الحالية بأسعار الفترة السابقة باستخدام كميات الفترة السابقة كأوزان، بينما يُحسب مؤشر باش باستخدام كميات الفترة الحالية كأوزان، ثم تُربط القيم السنوية للحصول على مؤشر تراكمي يعكس التغير الحقيقي عبر الزمن دون الاعتماد على سنة أساس ثابتة.

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Linking of Index Numbers

Linking means:
Converting several index numbers with different base years into one single consistent series with one base year.

That is:
We may have indices calculated with different base years such as:

- 2010 as base
- 2015 as base
- 2020 as base

We want to:
Link them into one continuous series.

Why do we need linking?

Because:

1. The base year becomes old.
2. Goods change (new goods appear, others disappear).
3. Weights become unrealistic.
4. Long time series become distorted.

So, linking is necessary in order to:
Maintain continuity of the time series.

Simple Linking Method (Direct Linking)

General formula

$$I_{t0} = \frac{I_{t0}}{I_{t-1}} \ , I_{t1} = \frac{I_{t1}}{I_{t-2}} \ , \ I_{t2} = \frac{I_{t2}}{I_{t-3}}$$

$$I_{t,t2} = I_{t0} * I_{t1} * I_{t2} * 100$$

That is: each year is linked to the previous year.

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Year	Index
2020	100
2021	120
2022	130
2023	140

- 2021/2020 = 120
- 2022/2021 = 108.3
- 2023/2022 = 107.7

linked

$I_{20,23} = 100 * 1.2 * 1.083 * 1.077 = 140$

Linking Two Series with Different Base Years

This is the most important case in exams.

We have:

- A first series with **2010 as base year**.
- A second series with **2015 as base year**.

We want to unify them into one series.

Numerical example:
First series (base year 2010)

Year	Index
2010	100
2011	110
2012	125
2013	140
2014	150

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Year	Index
2015	160

Second series (base year 2015)

Year	Index
2015	100
2016	108
2017	115
2018	130

Linking step:
We take the common year = **2015**.
Linking factor.

K=160/100=1.6

We multiply the entire second series by 1.6

Year	New Index
2015	160
2016	172.8
2017	184
2018	208