Example 8–28

Find the P-value when $\chi^2 = 3.823$, $n = 13$, and the test is left-tailed.

Solution

To get the P-value, look across the row with d.f. $= 12$ and find the two values that 3.823 falls between. They are 3.571 and 4.404. Look up to the top row and find the values corresponding to 3.571 and 4.404. They are 0.99 and 0.975, respectively. When the χ^2 test value falls on the left side, each of the values must be subtracted from 1 to get the interval that P-value falls between.

$$1 - 0.99 = 0.01 \qquad \text{and} \qquad 1 - 0.975 = 0.025$$

Hence the P-value falls in the interval

$$0.01 < P\text{-value} < 0.025$$

(The P-value obtained from a calculator is 0.014.)

When the χ^2 test is two-tailed, both interval values must be doubled. If a two-tailed test were being used in Example 8–28, then the interval would be $2(0.01) < P\text{-value} < 2(0.025)$, or $0.02 < P\text{-value} < 0.05$.

The P-value method for hypothesis testing for a variance or standard deviation follows the same steps shown in the preceding sections.

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

Step 3 Find the P-value.

Step 4 Make the decision.

Step 5 Summarize the results.

Example 8–29 shows the P-value method for variances or standard deviations.

Example 8–29

Car Inspection Times

A researcher knows from past studies that the standard deviation of the time it takes to inspect a car is 16.8 minutes. A sample of 24 cars is selected and inspected. The standard deviation is 12.5 minutes. At $\alpha = 0.05$, can it be concluded that the standard deviation has changed? Use the P-value method.

Solution

Step 1 State the hypotheses and identify the claim.

H_0: $\sigma = 16.8$ and H_1: $\sigma \neq 16.8$ (claim)

Step 2 Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(24-1)(12.5)^2}{(16.8)^2} = 12.733$$

Step 3 Find the P-value. Using Table G with d.f. $= 23$, the value 12.733 falls between 11.689 and 13.091, corresponding to 0.975 and 0.95, respectively. Since these values are found on the left side of the distribution, each value must be subtracted from 1. Hence $1 - 0.975 = 0.025$ and $1 - 0.95 = 0.05$. Since this is a two-tailed test, the area must be doubled to obtain the P-value interval. Hence $0.05 < P\text{-value} < 0.10$, or somewhere between 0.05 and 0.10. (The P-value obtained from a calculator is 0.085.)

The result will be shown in the first row of C2, 0.0168057. Since the P-value is less than α, reject the null hypothesis. The standard deviation in the sample is 11.2, the point estimate for the true standard deviation σ.

TI-83 Plus or TI-84 Plus
Step by Step

The TI-83 Plus and TI-84 Plus do not have a built-in hypothesis test for the variance or standard deviation. However, the downloadable program named SDHYP is available on your CD and Online Learning Center. Follow the instructions with your CD for downloading the program.

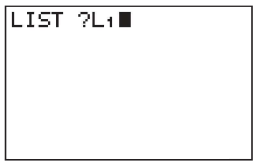
Performing a Hypothesis Test for the Variance and Standard Deviation (Data)

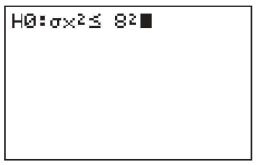
1. Enter the values into L_1.
2. Press **PRGM,** move the cursor to the program named SDHYP, and press **ENTER** twice.
3. Press **1** for Data.
4. Type $\mathbf{L_1}$ for the list and press **ENTER.**
5. Type the number corresponding to the type of alternative hypothesis.
6. Type the value of the hypothesized variance and press **ENTER.**
7. Press **ENTER** to clear the screen.

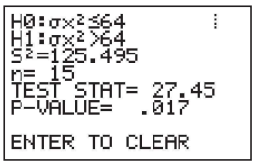
Example TI8–4

This pertains to Example 8–25 in the text. Test the claim that $\sigma > 8$ for these data.

25 30 5 15 18 42 16 9 10 12 12 38 8 14 27







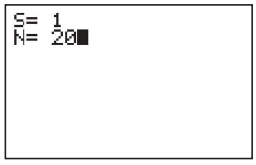
Since P-value $= 0.017 < 0.1$, we reject H_0 and conclude H_1. Therefore, there is enough evidence to support the claim that the standard deviation of the number of people using outpatient surgery is greater than 8.

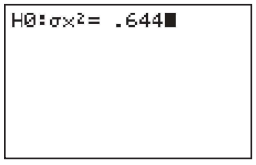
Performing a Hypothesis Test for the Variance and Standard Deviation (Statistics)

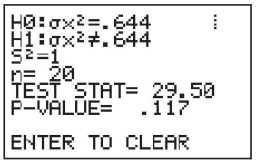
1. Press **PRGM,** move the cursor to the program named SDHYP, and press **ENTER** twice.
2. Press **2** for Stats.
3. Type the sample standard deviation and press **ENTER.**
4. Type the sample size and press **ENTER.**
5. Type the number corresponding to the type of alternative hypothesis.
6. Type the value of the hypothesized variance and press **ENTER.**
7. Press **ENTER** to clear the screen.

Example TI8–5

This pertains to Example 8–26 in the text. Test the claim that $\sigma^2 = 0.644$, given $n = 20$ and $s = 1$.







Since P-value $= 0.117 > 0.05$, we do not reject H_0 and do not conclude H_1. Therefore, there is not enough evidence to reject the manufacturer's claim that the variance of the nicotine content of the cigarettes is equal to 0.644.