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Theoretical Tests of Index Numbers

An index number is correct when it is based on clear scientific rules. Therefore, theoretical tests are used to check whether the index number is accurate and can measure the real change in prices or quantities.

One of the tests used to check the accuracy of index numbers is the **time reversal test**.

الاختبارات النظرية للأرقام القياسية يكون الرقم القياسي صحيحًا عندما يستند إلى قواعد علمية واضحة . ولذلك، تُستخدم الاختبارات النظرية للتحقق من دقة الرقم القياسي وقدرته على قياس التغير الحقيقي في الأسعار أو الكميات .ومن الاختبارات المستخدمة للتحقق من دقة الأرقام القياسية اختبار عكس الزمن.

1-Time reversal test

The **time reversal test** shows whether the index number gives a logical result when the time periods are reversed. If the new index is the **reciprocal of the original index**, that is, if the product of the two indices equals **1**, then the index number **passes the time reversal test**.

يُبين اختبار عكس الزمن ما إذا كان رقم القياسي يُعطي نتيجة منطقية عند عكس الفترات الزمنية .إذا كان رقم القياسي الجديد هو مقلوب الرقم القياسي الأصلي، أي إذا كان حاصل ضرب الرقمين يساوي 1، فإن رقم القياسي يجتاز اختبار عكس الزمن.

If we have the **simple aggregate price index**, then …

$$I = \left(\sum p_1 \Big/ \sum p_0\right) * 100$$

If we replace the current-period prices with the base-period prices $\left(\sum p_0 \Big/ \sum p_1\right)$, the time-reversed index is obtained, and the product of the two indices equals (1)

إذا استبدلنا أسعار الفترة الحالية بأسعار الفترة الأساسية

يتم الحصول على المؤشر المعكوس زمنيًا، ويكون حاصل ضرب المؤشرين مساويًا لـ (1).

$$\left(\sum p_1 \Big/ \sum p_0\right) * \left(\sum p_0 \Big/ \sum p_1\right) = 1$$

This indicates that the index number has passed the time reversal test.The same applies to the weighted aggregate price index using base-period quantities (Laspeyres index).

يشير هذا إلى أن الرقم القياسي قد اجتاز اختبار انعكاس الزمن. وينطبق الشيء نفسه على مؤشر الأسعار الإجمالي المرجح باستخدام كميات فترة الأساس (مؤشر لاسبير).

$$I_L = \left(\sum p_1\, q_0 \Big/ \sum p_0\, q_0\right) * 100$$

Its time-reversed index is $\left(\sum \boldsymbol{p_0\, q_1} \Big/ \sum \boldsymbol{p_1\, q_1}\right)$, and the product of the two indices is equal to …….

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$$\left(\sum p_1 q_0 / \sum p_0 q_0\right) * \left(\sum p_0 q_1 / \sum p_1 q_1\right) = 1$$

However, the Paasche index does not satisfy the time reversal test, as shown below. As for the Fisher index, …

$$\left(\sum p_1 q_1 / \sum p_0 q_1\right) * \left(\sum p_0 q_0 / \sum p_1 q_0\right) \neq 1$$

As for the Fisher index, …

$$I_F = \sqrt{\sum p_1 q_0 / \sum p_0 q_0 * \sum p_1 q_1 / \sum p_0 q_1} * \sqrt{\sum p_0 q_1 / \sum p_1 q_1 * \sum p_0 q_0 / \sum p_1 q_0} = 1$$

2-Factor Reversal Test

The **factor reversal test** is a theoretical test of index numbers used to check the **consistency between prices, quantities, and values**.

اختبار انعكاس العوامل

اختبار انعكاس العوامل هو اختبار نظري للأرقام القياسية يُستخدم للتحقق من التناسق بين الأسعار والكميات والقيم.

Basic idea:

This test is based on **replacing prices with quantities (or quantities with prices)** in the formula of the price index or quantity index, while keeping the **time reference unchanged**. By doing this, we obtain the corresponding **quantity index or price index**, which is called the **factor-reversed index**.

الفكرة الأساسية:

يعتمد هذا الاختبار على استبدال الأسعار بالكميات (أو الكميات بالأسعار) في معادلة الرقم القياسي السعر أو الكمية، مع الحفاظ على المرجع الزمني دون تغيير. وبذلك، نحصل على الرقم القياسي الكمي أو السعر المقابل، والذي يُسمى المؤشر المعكوس للعوامل.

If the **product of the price index and the quantity index is equal to the value index**, then the index number is said to **satisfy the factor reversal test**.

إذا كان حاصل ضرب مؤشر السعر ومؤشر الكمية يساوي مؤشر القيمة، فإن الرقم القياسي يقال إنه يفي باختبار انعكاس العامل.

1) Simple Aggregative Index

The **simple aggregative index** $\left(\sum p_1 / \sum p_0\right)$ and its **factor-reversed form** $\left(\sum q_1 / \sum q_0\right)$ are given as follows.

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When we multiply the simple aggregative index by its factor-reversed index, we find that the result **does not equal the value index**.

$$\left(\sum p_1 \big/ \sum p_0\right) * \left(\sum q_1 \big/ \sum q_0\right) \neq \left(\sum p_1 q_1 \big/ \sum p_0 q_0\right)$$

This means that the simple aggregative index **does not satisfy the factor reversal test**, and therefore, it **does not reflect the true change in prices**.

2) Quantity Indices (Laspeyres and Paasche)

The **Laspeyres and Paasche quantity indices** also **do not satisfy the factor reversal test**.

The product of the **Laspeyres index** $\sum p_1 q_0 \big/ \sum p_0 q_0$ and its factor-reversed index $\sum q_1 p_0 \big/ \sum q_0 p_0$ **is not equal** to the value index $\sum p_1 q_1 \big/ \sum p_0 q_0$.

$$\sum p_1 q_0 \big/ \sum p_0 q_0 * \sum q_1 p_0 \big/ \sum q_0 p_0 \neq \sum p_1 q_1 \big/ \sum p_0 q_0$$

Similarly, the product of the **Paasche index** and its factor-reversed index **is not equal** to the value index.

3) Fisher's Ideal Index

For the **ideal index (Fisher index)**, the product of the index and its factor-reversed form **equals the value index,** as shown below.

This means that the **Fisher index satisfies the factor reversal test**, and this is **one of the important reasons why it is called the ideal index**

$$\sqrt{\sum p_1 q_0 \big/ \sum p_0 q_0 * \sum p_1 q_1 \big/ \sum p_0 q_1} * \sqrt{\sum q_1 p_0 \big/ \sum q_0 p_0 * \sum q_1 p_1 \big/ \sum q_0 p_1}$$

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Below is an **exam-style numerical question** that covers **both** the **Time Reversal Test** and the **Factor (Coefficient) Reversal Test**, using **Laspeyres, Paasche, and Fisher**—with a **full solution**.

Numerical Question

For two commodities, the prices and quantities are:

Item	(p_0)	(q_0)	(p_1)	(q_1)
A	2	10	3	8
B	5	4	6	6

Required:

1. Compute the **Laspeyres price index** I_L, **Paasche price index** I_P, and **Fisher price index** I_F.
2. Compute the corresponding **quantity indices** Q_L, Q_P and Q_F.
3. Test the **Time Reversal Test** for I_L, I_P, and I_F.
4. Test the **Factor (Coefficient) Reversal Test** for Fisher (and show why Laspeyres/Paasche fail).

Solution

1) Price Indices

(a) Laspeyres Price Index

$$I_L = \left(\sum p_1\, q_0 \Big/ \sum p_0\, q_0\right) * 100$$

$$\sum p_1 q_0 = 3(10) + 6(4) = 30 + 24 = 54$$

$$\sum p_0 q_0 = 2(10) + 5(4) = 20 + 20 = 40$$

$$I_L = \left(54/40\right) * 100 = 135$$

(b) Paasche Price Index

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$$I_P = \left(\sum p_1\, q_1 \big/ \sum p_0\, q_1\right) * 100$$

$$\sum p_1 q_1 = 3(8) + 6(6) = 24 + 36 = 60$$

$$\sum p_0 q_1 = 2(8) + 5(6) = 16 + 30 = 46$$

$$I_P = \left(60/46\right) * 100 = 130.435$$

(c) Fisher Price Index

$$P_F = \sqrt{P_L \times P_P} = \sqrt{1.35 \times 1.30435} = \sqrt{1.76087} = 1.327 \quad (132.7)$$

2) Quantity Indices

(a) Laspeyres Quantity Index

$$Q_L = \sum q_0\, p_1 \big/ \sum q_1\, p_1 * 100 = \frac{46}{40} = 115$$

(b) Paasche Quantity Index

Paasche quantity index

$$Q_P = \frac{\sum p_1 q_1}{\sum p_1 q_0} = \frac{60}{54} = 1.1111$$

$$Q_P = 111.11$$

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Fisher quantity index

$$Q_F = \sqrt{Q_L \times Q_P} = \sqrt{1.15 \times 1.1111} = 1.1304$$

$$Q_F = 113.04$$

3-Time Reversal Test for IL, IP, and IF

Recall the price indices:

Laspeyres price index

$$I_L = \frac{\sum p_1 q_0}{\sum p_0 q_0} = \frac{54}{40} = 1.35$$

Paasche price index

$$I_P = \frac{\sum p_1 q_1}{\sum p_0 q_1} = \frac{60}{46} = 1.30435$$

Fisher price index

$$I_F = \sqrt{I_L \times I_P} = \sqrt{1.35 \times 1.30435} = 1.32698$$

The Time Reversal Test requires:

$$P_{01} \times P_{10} = 1$$

(a) Laspeyres

Reverse Laspeyres:

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$$I_L^{10} = \frac{\sum p_0 q_1}{\sum p_1 q_1} = \frac{46}{60} = 0.76667$$

Then:

$$I_L^{01} \times I_L^{10} = 1.35 \times 0.76667 = 1.035$$

Since:

$$1.035 \neq 1$$

Laspeyres **fails** the Time Reversal Test.

(b) Paasche

Reverse Paasche:

$$I_P^{10} = \frac{\sum p_0 q_0}{\sum p_1 q_0} = \frac{40}{54} = 0.74074$$

Then:

$$I_P^{01} \times I_P^{10} = 1.30435 \times 0.74074 = 0.96618$$

Since:

$$0.96618 \neq 1$$

Paasche **fails** the Time Reversal Test.

(c) Fisher

Reverse Fisher:

$$I_F^{10} = \sqrt{I_L^{10} \times I_P^{10}} = \sqrt{0.76667 \times 0.74074} = 0.75359$$

Then:

$$I_F^{01} \times I_F^{10} = 1.32698 \times 0.75359 = 1$$

So Fisher **satisfies** the Time Reversal Test.

Conclusion

- IL: fails
- IP: fails
- IF: passes

4) Factor (Coefficient) Reversal Test

The Factor Reversal Test requires:

$$P_{01} \times Q_{01} = V_{01}$$

where

$$V_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = \frac{60}{40} = 1.5$$

Fisher index

$$I_F \times Q_F = 1.32698 \times 1.1304 = 1.5$$

And since:

$$V_{01} = 1.5$$

therefore Fisher **satisfies** the Factor Reversal Test.

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In percentage form:

$$\frac{132.70 \times 113.04}{100} = 150$$

which equals the value index 150150150.

Why Laspeyres fails

$$I_L \times Q_L = 1.35 \times 1.15 = 1.5525$$

But:

$$V_{01} = 1.5$$

Since:

$$1.5525 \neq 1.5$$

Laspeyres **fails** the Factor Reversal Test.

Why Paasche fails

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$$I_P \times Q_P = 1.30435 \times 1.1111 = 1.44928$$

But:

$$V_{01} = 1.5$$

Since:

$$1.44928 \neq 1.5$$

Paasche **fails** the Factor Reversal Test.

Final summary

$$Q_L = 115, \qquad Q_P = 111.11, \qquad Q_F = 113.04$$

Time Reversal Test

- Laspeyres: **fails**
- Paasche: **fails**
- Fisher: **passes**

Factor Reversal Test

- Fisher: **passes**
- Laspeyres: **fails**
- Paasche: **fails**