Measure the General non-linear trend
قياس الاتجاه العام غير الخطي

There are some phenomena that are subject to the general non-linear trend, i.e. the data of these phenomena take different non-linear forms, and the non-linear forms that will be discussed are:

1) Curves of the second and third order.
2) Exponential equation

The curves are of the second and third order **المنحنيات من الدرجة الثانية والثالثة**

There is no doubt that fitting the general trend line using the least squares method leads to more accurate results compared to all the methods previously mentioned, but the accuracy of its results depends to a large extent on determining the general trend of the graph that suits the development of the phenomenon in the period under study, i.e. on determining the degree of the equation that is suitable for implementing the general trend of the phenomenon under study, as the general trend that the phenomenon takes may be more consistent with the equation of the second-degree curve (parabola), and the form of the second-degree equation is:

$$\hat{y} = a + bt + ct^2$$

Where:

$\hat{y}$: represents the estimated value of the phenomenon. تمثل القيمة التقديرية للظاهرة.

a, b, c: are constants.

لاشك ان توفيق خط الاتجاه العام بطريقة المربعات الصغرى يؤدي الى نتائج اكثر دقة بالمقارنة مع جميع الطرائق التي سبق الاشارة اليها الا ان الدقة نتائجها تتوقف الى حد كبير على تحديد الاتجاه العام للخط البياني الذي يناسب تطور الظاهرة في الفترة موضوع الدراسة اي على تحديد درجة المعادلة التي تصلح لتنفيذ الاتجاه العام للظاهرة موضوع الدراسة اذ قد يكون الاتجاه العام الذي تأخذه الظاهرة يتفق اكثر مع معادلة المنحني من الدرجة الثانية (القطع المكافئ)

The equation of the second-degree curve can be estimated using the least squares method by estimating the coefficients (a, b, c) through the necessary natural equations, which are:

$\hat{y} = a + bt + ct^2$ ... (أ)

$\sum y_i = na + b\sum t_i + c\sum t_i^2$ ... (1) بضرب المعادلة (أ) بـ $\sum$ نحصل على:

$\sum t_i y_i = a\sum t_i + b\sum t_i^2 + c\sum t_i^3$ ... (2) بضرب المعادلة (أ) بـ $\sum t_i$ نحصل على:

$\sum t_i^2 y_i = a\sum t_i^2 + b\sum t_i^3 + c\sum t_i^4 \quad \ldots (3)$ بضرب المعادلة (أ) بـ $\sum t_i^2$ نحصل على:

وبحل المعادلات الثلاثة انياً نحصل على قيم (a,b,c).

To simplify these equations, we use the same concepts mentioned in the first-degree equation with a change in the origin point, which depends on the number of years, even or odd, at the midpoint of the time period of the series, where it is (zero) as the origin point. Thus, the three equations become as follows:

$$\sum y_i = na + b(0) + c\sum t_i^2$$

$$\sum y_i = na + c\sum t_i^2 \quad \ldots (1)$$

$$\sum t_i y_i = a(0) + b\sum t_i^2 + c(0)$$

$$\sum t_i y_i = b\sum t_i^2$$

$$b = \frac{\sum t_i y_i}{\sum t_i^2} \quad \ldots (2)$$

$$\sum t_i^2 y_i = a\sum t_i^2 + b(0) + c\sum t_i^4$$

$$\sum t_i^2 y_i = a\sum t_i^2 + c\sum t_i^4 \quad \ldots (3)$$

The second degree equation does not give a specific general direction, as the curve means that the rate of change varies from one point to another. To know the directional value of any of the years included in the time series, it is only affected by its general direction. We replace (t) in the general direction equation with what is equal to it, according to the order, and we obtain the rate of change in this year.

Example: The following data represent a time series of annual fertilizer production for the period 1970-1980. The required equation is to be estimated from the second degree and the production forecast for the year 1981.

Years	Production	t	t*y	t^2	t^2*y	t^4
1970	250	-5	-1250	25	6250	625
71	295	-4	-1180	16	4720	256
72	334	-3	-1002	9	3006	81
73	350	-2	-700	4	1400	16
74	361	-1	-361	1	361	1
75	373	0	0	0	0	0
76	362	1	362	1	362	1
77	349	2	698	4	1396	16
78	341	3	1023	9	3069	81
79	339	4	1356	16	5424	256
80	334	5	1670	25	8350	625
Sum	$\sum_{i=1}^{n} y_i = 3688$	$\sum_{i=1}^{n} t_i = 0$	$\sum_{i=1}^{n} y_i t_i = 616$	$\sum_{i=1}^{n} t_i^2 = 110$	$\sum_{i=1}^{n} y_i t_i^2 = 34338$	$\sum_{i=1}^{n} t_i^2 = 1958$

$\hat{y} = a + bt + ct^2$

$$b = \frac{\sum t_i y_i}{\sum t_i^2} = \frac{616}{110} = 5.6$$

$$\sum y_i = na + c\sum t_i^2 \quad \ldots (1)$$

$$\sum t_i^2 y_i = a\sum t_i^2 + c\sum t_i^4 \quad \ldots (2)$$

3688 = 11(a) + 110(c) …(1)

34338 = 110(a) + 1958(c) …(2)

By multiplying equation (1) by the number (10), we get:

36880 = 110(a) + 1100(c) …(1)

34338 = 110(a) + 1958(c) …(2)

بالطرح

2542 = −858(c) ⟶ c = −2542/858 = −2.963

By substituting the value of (c) in equation (1), we get the value of (a).

3688 = 11(a) + 110(c)

3688 = 11(a) + 110(−2.963)

a=364.9

$\hat{y} = 364.9 + 5.6t - 2.963t^2$

$\hat{y}_{1981} = 364.9 + 5.6(6) - 2.963(6)^2 = 291.832$