Solution

Now H_0: $\mu = 5$ and H_1: $\mu \neq 5$ (claim). The critical values are $+2.010$ and -2.010. The test value is

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{4.6 - 5.0}{0.7/\sqrt{50}} = \frac{-0.4}{0.099} = -4.04$$

Since $-4.04 < -2.010$, the null hypothesis is rejected. There is enough evidence to support the claim that the bags do not weigh 5 pounds.

The 95% confidence for the mean is given by

$$\bar{X} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{\alpha/2}\frac{s}{\sqrt{n}}$$

$$4.6 - (2.010)\left(\frac{0.7}{\sqrt{50}}\right) < \mu < 4.6 + (2.010)\left(\frac{0.7}{\sqrt{50}}\right)$$

$$4.4 < \mu < 4.8$$

Notice that the 95% confidence interval of μ does *not* contain the hypothesized value $\mu = 5$. Hence, there is agreement between the hypothesis test and the confidence interval.

Example 8–31 Hog Weights

A researcher claims that adult hogs fed a special diet will have an average weight of 200 pounds. A sample of 10 hogs has an average weight of 198.2 pounds and a standard deviation of 3.3 pounds. At $\alpha = 0.05$, can the claim be rejected? Also, find the 95% confidence interval of the true mean.

Solution

Now H_0: $\mu = 200$ pounds (claim) and H_1: $\mu \neq 200$ pounds. The t test must be used since σ is unknown. It is assumed that hog weights are normally distributed. The critical values at $\alpha = 0.05$ with 9 degrees of freedom are $+2.262$ and -2.262. The test value is

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{198.2 - 200}{3.3/\sqrt{10}} = \frac{-1.8}{1.0436} = -1.72$$

Thus, the null hypothesis is not rejected. There is not enough evidence to reject the claim that the weight of the adult hogs is 200 pounds.

The 95% confidence interval of the mean is

$$\bar{X} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{\alpha/2}\frac{s}{\sqrt{n}}$$

$$198.2 - (2.262)\left(\frac{3.3}{\sqrt{10}}\right) < \mu < 198.2 + (2.262)\left(\frac{3.3}{\sqrt{10}}\right)$$

$$198.2 - 2.361 < \mu < 198.2 + 2.361$$

$$195.8 < \mu < 200.6$$

The 95% confidence interval does contain the hypothesized mean $\mu = 200$. Again there is agreement between the hypothesis test and the confidence interval.

In summary, then, when the null hypothesis is rejected at a significance level of α, the confidence interval computed at the $1 - \alpha$ level will not contain the value of the mean

The standard deviation of the population was 3 minutes. To see whether the average time had changed since the newspaper's format was revised, the newspaper editor surveyed 36 individuals. The average time that the 36 people spent reading the paper was 12.2 minutes. At $\alpha = 0.02$, is there a change in the average time an individual spends reading the newspaper? Find the 98% confidence interval of the mean. Do the results agree? Explain.

7. What is meant by the power of a test?
8. How is the power of a test related to the type II error?
9. How can the power of a test be increased?

Summary

This chapter introduces the basic concepts of hypothesis testing. A statistical hypothesis is a conjecture about a population. There are two types of statistical hypotheses: the null and the alternative hypotheses. The null hypothesis states that there is no difference, and the alternative hypothesis specifies a difference. To test the null hypothesis, researchers use a statistical test. Many test values are computed by using

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

- Researchers compute a test value from the sample data to decide whether the null hypothesis should be rejected. Statistical tests can be one-tailed or two-tailed, depending on the hypotheses.

  The null hypothesis is rejected when the difference between the population parameter and the sample statistic is said to be significant. The difference is significant when the test value falls in the critical region of the distribution. The critical region is determined by α, the level of significance of the test. The level is the probability of committing a type I error. This error occurs when the null hypothesis is rejected when it is true. Three generally agreed upon significance levels are 0.10, 0.05, and 0.01. A second kind of error, the type II error, can occur when the null hypothesis is not rejected when it is false. (8–1)
- There are two common methods used to test hypotheses; they are the traditional method and the *P*-value method. (8–2)
- All hypothesis-testing situations using the traditional method should include the following steps:

  1. State the null and alternative hypotheses and identify the claim.
  2. State an alpha level and find the critical value(s).
  3. Compute the test value.
  4. Make the decision to reject or not reject the null hypothesis.
  5. Summarize the results.
- All hypothesis-testing situations using the *P*-value method should include the following steps:

  1. State the hypotheses and identify the claim.
  2. Compute the test value.
  3. Find the *P*-value.
  4. Make the decision.
  5. Summarize the results.
- The *z* test is used to test a mean when the population standard deviation is known. When the sample size is less than 30, the population values need to be normally distributed. (8–2)