Measure the General non-linear trend
قياس الاتجاه العام غير الخطي

2) Exponential Equation المعادلة الاسية

It is sometimes called the semi-logarithmic equation. **The general exponential trend** is defined as the general trend in which the value of the phenomenon (y) does not increase by a fixed absolute amount for each year, but rather by fixed percentages for each year. In other words, when the dependent variable (y) changes at a certain rate, that is according to a certain geometric sequence. The exponential equation takes the following form:

$$y_i = ab^t$$

Where:
y_i: represents the value of the phenomenon at time i.
a, b : are constants.
t: represents the independent variable.

وتسمى في بعض الاحيان بالمعادلة نصف اللوغارتمية **ويعرف الاتجاه العام الأسي** بانه عبارة عن الاتجاه العام الذي لا تزداد فيه قيمة الظاهرة (y) بكمية ثابتة مطلقة لكل سنة بل بنسب مئوية ثابتة لكل سنة وبعبارة اخرى عندما يتغير المتغير التابع (y) وفق معدل معين اي وفق متوالية هندسية معينة

The exponential equation above can be converted to a linear equation which is more useful than putting it in exponential form after entering the logarithm on it as follows:

$$\log(y_i) = \log(a) + t\, log(b) \qquad \ldots (*)$$

In the above equation, the variable (t) is still a sequence and has not been transformed into (log(t)). That is why this function is called the semilogarithmic function. To estimate the exponential equation in its linear form, (log(b), log(a)) must be estimated. This leads us to forming the necessary natural equations.
By multiplying equation (*) by $\sum$ we get:

$$\sum \log(y_i) = n \log(a) + log(b) \sum t_i \qquad \ldots (1)$$

By multiplying equation (*) by $\sum t_i$ we get:

$$\sum t_i \log(y_i) = \log(a) \sum t_i + log(b) \sum t_i^2 \qquad \dots (2)$$

By substituting in the two equations and solving them simultaneously, we find the value of each of (log(b), log(a)) and from there we find (a, b) and substitute them in the exponential equation where the value of (a, b) is found by taking the corresponding number (10^t) for each of (log(b), log(a)).

بالتعويض في المعادلتين وحلهما انياً نجد قيمة كل من $(log(b), log(a))$ ومنها نجد (a,b) ونعوضهما في المعادلة الاسية حيث يتم ايجاد قيمة (a,b) من خلال اخذ العدد المقابل (10^t) لكل من $(log(b), log(a))$.

The arithmetic operations can also be simplified in finding the values of (a, b) by moving the origin point with respect to time to its value from the intermediate values, then:

$$\sum \log(y_i) = n \log(a) + log(b)\ (0)$$

$$\log(a) = \frac{\sum \log(y_i)}{n} \qquad \dots (3)$$

$$\sum t_i \log(y_i) = \log(a)\ (0) + log(b) \sum t_i^2$$

$$log(b) = \frac{\sum t_i \log(y_i)}{\sum t_i^2} \qquad \dots (4)$$

Example: The following data represents the sales of a company. The required equation is to find the exponential curve equation (semi-logarithmic equation) and forecast sales for the year 1990.

Years	Sales	**t**	t^2	**Log(y)**	**t log(y)**
1985	1	-2	4	0	0
1986	3	-1	1	0.477	-0.477
1987	6	0	0	0.778	0
1988	14	1	1	1.146	1.146
1989	41	2	4	1.613	3.226
Sum	$\sum_{i=1}^{n} y_i = 65$	$\sum_{i=1}^{n} t_i = 0$	$\sum_{i=1}^{n} t_i^2 = 10$	$\sum_{i=1}^{n} log(y_i) = 4.014$	$\sum t_i log(y_i) = 3.895$

$y_i = ab^t$

$\log(y_i) = \log(a) + t\ log(b)$

$$\log(a) = \frac{\sum \log(y_i)}{n} = \frac{4.014}{5} = 0.8028$$

$$log(b) = \frac{\sum t_i \log(y_i)}{\sum t_i^2} = \frac{3.895}{10} = 0.3895$$

$\log(y_i) = 0.8028 + t\ (0.3895)$

بأخذ العدد المقابل (10^t) لكل من ($log(b), log(a)$) نحصل على:

a=6.35 b=2.45

$\hat{y}_i = (6.35)(2.45)^t$

$y_{1990} = (6.35)(2.45)^3 = 93.384$