Biostatistics – Spring 2026
Lecture 06: Population Estimation Methods
Arithmetic and Geometric Growth

Dr. Zaid T. Al-Khaledi
Department of Statistics and Informatics
University of Mosul

March 18, 2026

Introduction

In public health and demography, many important measures depend on the population size. For example, when we calculate prevalence, incidence, birth rate, or mortality rate, the denominator usually comes from the number of people in the population.

But in practice, the exact population size is not always available for every year. Therefore, we often need to estimate the population between two census years.

This lecture introduces two standard methods:

- Arithmetic growth method
- Geometric growth method

These methods are simple and very useful in demographic work.

By the end of this lecture, you should be able to:

- Explain why population estimation is important in biostatistics and public health.
- Use the arithmetic growth formula to estimate population.
- Use the geometric growth formula to estimate population.
- Compare the two methods.
- Give a demographic interpretation of the estimated values.

1. Why Do We Need Population Estimation?

Many vital and health measures are rates, and rates need a denominator.

Examples:

- Crude birth rate
- Crude mortality rate
- Incidence rate
- Prevalence proportion

If the denominator is wrong, the rate will also be wrong.

Practical problem

Suppose census data are available only in 1981 and 1990, but we want the population in 1985. Since 1985 is between the two known years, we estimate the population using a suitable growth model.

2. Arithmetic Growth Method

The arithmetic growth method assumes that the population changes by a constant numerical amount each year.

In other words, the increase from one year to the next is assumed to be the same.

2.1 Formula

$$P_t = P_1 + \frac{t}{n}(P_n - P_1)$$

where:

- P_t = estimated population at time t
- P_1 = population at the beginning of the period
- P_n = population at the end of the period
- n = total number of years in the period
- t = number of years from the first year to the target year

2.2 Interpretation

This method assumes a straight-line increase or decrease.

So if the population increases by 2,000 people per year, it keeps increasing by 2,000 every year.

3. Geometric Growth Method

The geometric growth method assumes that the population grows by a constant **percentage** rather than a constant numerical amount.

This is often more realistic because population growth usually affects the following years.

3.1 Formula

$$P_t = \left(\frac{P_n}{P_1}\right)^{\frac{t}{n}} \times P_1$$

This can also be written in logarithmic form:

$$\log P_t = \frac{t}{n}(\log P_n - \log P_1) + \log P_1$$

3.2 Interpretation

This method assumes compound growth.

That means the increase in one year becomes part of the base for the next year.

4. Example: Estimating the Population in 1985

Suppose the population of a city was:

$$P_1 = 100{,}000 \quad \text{in 1981}$$

and

$$P_n = 120{,}000 \quad \text{in 1990.}$$

Estimate the population in 1985.

Step 1: Determine t and n

From 1981 to 1990, the full period is taken as 10 years in this teaching example.

The target year 1985 corresponds to:

$$t = 5, \qquad n = 10.$$

4.1 Arithmetic growth solution

$$P_t = P_1 + \frac{t}{n}(P_n - P_1)$$

$$P_5 = 100{,}000 + \frac{5}{10}(120{,}000 - 100{,}000)$$

$$P_5 = 100{,}000 + \frac{5}{10}(20{,}000)$$

$$P_5 = 100{,}000 + 10{,}000 = 110{,}000.$$

Interpretation

Using arithmetic growth, the estimated population in 1985 is 110,000.

4.2 Geometric growth solution

$$\log P_t = \frac{t}{n}(\log P_n - \log P_1) + \log P_1$$

Using common logarithms:

$$\log 120{,}000 \approx 5.0792, \qquad \log 100{,}000 = 5.$$

So,

$$\log P_5 = \frac{5}{10}(5.0792 - 5) + 5$$

$$= \frac{1}{2}(0.0792) + 5$$

$$= 0.0396 + 5 = 5.0396.$$

Taking antilog:

$$P_5 \approx 109{,}547.$$

Interpretation

Using geometric growth, the estimated population in 1985 is approximately 109,547.

5. Comparison Between Arithmetic and Geometric Methods

In this example, we obtained:

$$\text{Arithmetic estimate} = 110{,}000$$
$$\text{Geometric estimate} \approx 109{,}547.$$

The two results are close, but not identical.

Why are they different?

Because the assumptions are different.

- Arithmetic method assumes a constant absolute increase.
- Geometric method assumes a constant relative (percentage) increase.

General rule

If population change is approximately linear, arithmetic growth may be acceptable.

If population growth behaves like compound increase, geometric growth is often more realistic.

6. Demographic Interpretation

Population estimation is not only a mathematical exercise. It has practical demographic meaning.

6.1 Why does it matter?

Suppose we want to compute the crude death rate in 1985. We need the population denominator. If that denominator is too large or too small, the death rate will be misleading.

6.2 Example of demographic use

If 880 deaths occurred in 1985 and the estimated population is 110,000, then

$$\text{Crude death rate} = \frac{880}{110{,}000} \times 1000 = 8$$

So the crude death rate is 8 deaths per 1000 population.
If a different population estimate is used, the final rate will change slightly.

6.3 Important interpretation

When reporting population-based rates, always remember:

> The numerator may come from medical records, but the denominator often comes from census or estimated population data.

Therefore, good population estimation is essential in epidemiology and public health.

7. Another Example

Suppose a town had:

$$P_1 = 50{,}000 \quad \text{in } 2010$$

and

$$P_n = 65{,}000 \quad \text{in } 2020.$$

Estimate the population in 2016 using both methods.
Here,

$$t = 6, \qquad n = 10.$$

Arithmetic method

$$P_t = 50{,}000 + \frac{6}{10}(65{,}000 - 50{,}000)$$

$$= 50{,}000 + \frac{6}{10}(15{,}000)$$

$$= 50{,}000 + 9{,}000 = 59{,}000.$$

Geometric method

$$P_t = \left(\frac{65{,}000}{50{,}000}\right)^{\frac{6}{10}} \times 50{,}000$$

$$= (1.3)^{0.6} \times 50{,}000$$

$$\approx 1.1704 \times 50{,}000$$

$$\approx 58{,}520.$$

Interpretation

Again, the two estimates are close, but the arithmetic method gives a slightly larger value in this example.

8. Common Student Mistakes

- Confusing constant amount growth with constant percentage growth.
- Using the wrong value of t.
- Forgetting that n is the total period length.
- Reporting the estimated population without interpretation.
- Using the result without explaining why the denominator matters in rate calculation.

9. Summary of the Lecture

- Population estimation is important because many biostatistical measures need a denominator.
- Arithmetic growth assumes constant numerical increase.
- Geometric growth assumes constant percentage increase.
- The two methods may give slightly different estimates.
- The choice of method affects demographic and health rate calculations.

Homework (HW)

HW1

The population of a city was 200,000 in 2012 and 260,000 in 2022.
Estimate the population in 2017 using:

1. The arithmetic growth method.
2. The geometric growth method.
3. Compare the two results.

HW2

A district had population 80,000 in 2015 and 92,000 in 2021.
Estimate the population in 2018 using both methods.
Then answer:

1. Which estimate is larger?
2. Why might the two methods differ?

HW3

In a town, 540 deaths were recorded in 2019. The population was 70,000 in 2015 and 82,000 in 2025.
Required:

1. Estimate the population in 2019 using the arithmetic method.
2. Estimate the population in 2019 using the geometric method.

3. Compute the crude death rate per 1000 using each estimate.
4. Comment on the effect of the denominator.