Step 2 Find the critical values. Since $\alpha = 0.05$, the critical values are $+1.96$ and -1.96.

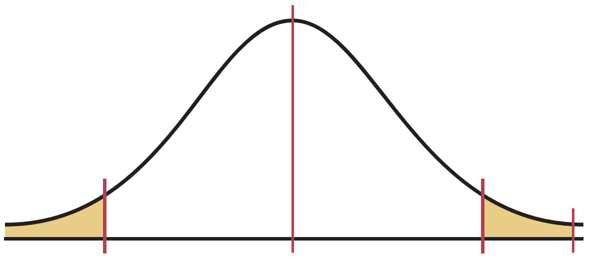
Step 3 Compute the test value.

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(88.42 - 80.61) - 0}{\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}}} = 7.45$$

Step 4 Make the decision. Reject the null hypothesis at $\alpha = 0.05$, since $7.45 > 1.96$. See Figure 9–3.

Figure 9–3

Critical and Test Values for Example 9–1



Step 5 Summarize the results. There is enough evidence to support the claim that the means are not equal. Hence, there is a significant difference in the rates.

The *P*-values for this test can be determined by using the same procedure shown in Section 8–2. For example, if the test value for a two-tailed test is 1.40, then the *P*-value obtained from Table E is 0.1616. This value is obtained by looking up the area for $z = 1.40$, which is 0.9192. Then 0.9192 is subtracted from 1.0000 to get 0.0808. Finally, this value is doubled to get 0.1616 since the test is two-tailed. If $\alpha = 0.05$, the decision would be to not reject the null hypothesis, since *P*-value $> \alpha$.

The *P*-value method for hypothesis testing for this chapter also follows the same format as stated in Chapter 8. The steps are reviewed here.

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

Step 3 Find the *P*-value.

Step 4 Make the decision.

Step 5 Summarize the results.

Example 9–2 illustrates these steps.

Example 9–2 College Sports Offerings

A researcher hypothesizes that the average number of sports that colleges offer for males is greater than the average number of sports that colleges offer for females. A sample of the number of sports offered by colleges is shown. At $\alpha = 0.10$, is there enough evidence to support the claim? Assume σ_1 and $\sigma_2 = 3.3$.

community college are, on average, 3.2 years older than those at a university. In this case, the hypotheses are

$$H_0: \mu_1 - \mu_2 = 3.2 \qquad \text{and} \qquad H_1: \mu_1 - \mu_2 > 3.2$$

The formula for the z test is still

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

where $\mu_1 - \mu_2$ is the hypothesized difference or expected value. In this case, $\mu_1 - \mu_2 = 3.2$.

Confidence intervals for the difference between two means can also be found. When you are hypothesizing a difference of zero, if the confidence interval contains zero, the null hypothesis is not rejected. If the confidence interval does not contain zero, the null hypothesis is rejected.

Confidence intervals for the difference between two means can be found by using this formula:

Formula for the *z* Confidence Interval for Difference Between Two Means

$$(\bar{X}_1 - \bar{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Example 9–3

Find the 95% confidence interval for the difference between the means for the data in Example 9–1.

Solution

Substitute in the formula, using $z_{\alpha/2} = 1.96$.

$$(\bar{X}_1 - \bar{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2$$

$$< (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(88.42 - 80.61) - 1.96\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}} < \mu_1 - \mu_2$$

$$< (88.42 - 80.61) + 1.96\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}}$$

$$7.81 - 2.05 < \mu_1 - \mu_2 < 7.81 + 2.05$$

$$5.76 < \mu_1 - \mu_2 < 9.86$$

Since the confidence interval does not contain zero, the decision is to reject the null hypothesis, which agrees with the previous result.

Applying the Concepts 9–1

Home Runs

For a sports radio talk show, you are asked to research the question whether more home runs are hit by players in the National League or by players in the American League. You decide to use the home run leaders from each league for a 40-year period as your data. The numbers are shown.

6. **Teachers' Salaries** California and New York lead the list of average teachers' salaries. The California yearly average is $64,421 while teachers in New York make an average annual salary of $62,332. Random samples of 45 teachers from each state yielded the following.

	California	New York
Sample mean	64,510	62,900
Population standard deviation	8,200	7,800

At $\alpha = 0.10$ is there a difference in means of the salaries?

Source: *World Almanac.*

7. **Commuting Times** The Bureau of the Census reports that the average commuting time for citizens of both Baltimore, Maryland, and Miami, Florida, is approximately 29 minutes. To see if their commuting times appear to be any different in the winter, random samples of 40 drivers were surveyed in each city and the average commuting time for the month of January was calculated for both cities. The results are provided below. At the 0.05 level of significance, can it be concluded that the commuting times are different in the winter?

	Miami	Baltimore
Sample size	40	40
Sample mean	28.5 min	35.2 min
Population standard deviation	7.2 min	9.1 min

Source: www.census.gov

8. **Heights of 9-Year-Olds** At age 9 the average weight (21.3 kg) and the average height (124.5 cm) for both boys and girls are exactly the same. A random sample of 9-year-olds yielded these results. Estimate the mean difference in height between boys and girls with 95% confidence. Does your interval support the given claim?

	Boys	Girls
Sample size	60	50
Mean height, cm	123.5	126.2
Population variance	98	120

Source: www.healthepic.com

9. **Length of Hospital Stays** The average length of "short hospital stays" for men is slightly longer than that for women, 5.2 days versus 4.5 days. A random sample of recent hospital stays for both men and women revealed the following. At $\alpha = 0.01$, is there sufficient evidence to conclude that the average hospital stay for men is longer than the average hospital stay for women?

	Men	Women
Sample size	32	30
Sample mean	5.5 days	4.2 days
Population standard deviation	1.2 days	1.5 days

Source: www.cdc.gov/nchs

10. **Home Prices** A real estate agent compares the selling prices of homes in two municipalities in southwestern Pennsylvania to see if there is a difference. The results of the study are shown. Is there enough evidence to reject the claim that the average cost of a home in both locations is the same? Use $\alpha = 0.01$.

Scott	Ligonier
$\bar{X}_1 = \$93,430$*	$\bar{X}_2 = \$98,043$*
$\sigma_1 = \$5602$	$\sigma_2 = \$4731$
$n_1 = 35$	$n_2 = 40$

*Based on information from RealSTATs.

11. **Women Science Majors** In a study of women science majors, the following data were obtained on two groups, those who left their profession within a few months after graduation (leavers) and those who remained in their profession after they graduated (stayers). Test the claim that those who stayed had a higher science grade point average than those who left. Use $\alpha = 0.05$.

Leavers	Stayers
$\bar{X}_1 = 3.16$	$\bar{X}_2 = 3.28$
$\sigma_1 = 0.52$	$\sigma_2 = 0.46$
$n_1 = 103$	$n_2 = 225$

Source: Paula Rayman and Belle Brett, "Women Science Majors: What Makes a Difference in Persistence after Graduation?" *The Journal of Higher Education.*

12. **ACT Scores** A survey of 1000 students nationwide showed a mean ACT score of 21.4. A survey of 500 Ohio scores showed a mean of 20.8. If the population standard deviation in each case is 3, can we conclude that Ohio is below the national average? Use $\alpha = 0.05$.

Source: Report of WFIN radio.

13. **Per Capita Income** The average per capita income for Wisconsin is reported to be $37,314, and for South Dakota it is $37,375—almost the same thing. A random sample of 50 workers from each state indicated the following sample statistics.

	Wisconsin	South Dakota
Size	50	50
Mean	$40,275	$38,750
Population standard deviation	$10,500	$12,500

At $\alpha = 0.05$ can we conclude a difference in means of the personal incomes?

Source: *New York Times Almanac.*

14. **Monthly Social Security Benefits** The average monthly Social Security benefit in 2004 for retired workers was $954.90 and for disabled workers was $894.10. Researchers used data from the Social Security records to test the claim that the difference in monthly benefits between the two groups was greater than $30.