2.5 Problems

1. How many 3- digit numbers can be made from the 10 digits 0,1,2, ...,9? How many of these are odd? How many of these are divisible by 5?
2. There are 5 roads between city A and city B, and 4 roads between B and C.

i. In how many ways can one drive from city A to city C by way of B?

ii. In how many ways can one drive from A to C and back to C (passing through B on both trips without using the same road more than once?

3. In how many ways can a student arrange his 8 textbooks on a shelf? In how many ways can he do this if three specific books must be together?
4. Find n if (i) $P(n,3)=336$, (ii) $P(n,3)= 5P(n,2)$. (iii) $2P(n,2) + 50=P(2n,2)$.
5.

a. Compute $C(8,4)$, $C(13,3)$, $C(20,0)$ and $C(5,7)$.

b. Show that $\sum_{r=0}^{n} C(n,r) = 2^n$

6. Using Theorem 2.3.3 to show that $\sum_{r=0}^{n} (C(n,r))^2 = C(2n,n)$.
7. Mr. Ahmad has 5 suits, 7 ties and 4 pairs of shoes. In how many ways can he choose 3 suits, 4 ties and 2 pairs of shoes to take along on a business trip.
8. A student is to answer 7 questions out of 10 questions in an exam. How many choices has he (1) if he must answer the first 3 questions?(2) if he must not answer the last question.?
9. Mr. Ayad has 10 books that he is going to put on his book shelf. Of these, 4 are mathematics books, 3 are chemistry books, 2 are history books, and 1 is a language book. Ayad wants to arrange his books so that all the books dealing will the same subject are together on the shelf. How many different arrangements are possible?
10. If 4 Iraqis, 3 Egyptions and 2 Jordanians are to be seated

in a row. In how many ways can this be done if the people of the same nationality must sit together?

11. Solve problem 10, if they sit at a round table.

12. A class cantains 10 boys and 5 girls.
   i. In how many ways can a teacher choose a committee of 4?
   ii. How many of these committees will contain 2 girls?
   iii. How many of these committees will contain at least one boy?

13. A multiple-choice test consists of 6 questions, each permitting a choice of 4 altrenatives. In how many differen ways can a student check off his answer to these questions?

14. On each business trip, a salesman visits 5 of the major cities in his territory. In how many different ways can he schedule his route (that is the cities and their order) for such a trip?

15. A woman has 8 friends, of whom she will invite 5 to a tea party. How many chioces does she have if 2 of the friends are feuding and will not attend together? How many chioces does she have if 2 of them will only attend together?

16. The game of bridge is played by 4 players, each of whom is dealt 13 cards. How many bridge hands are possible?

17. In how many ways can a man divide 7 gifts among his 3 children if the eldest is to receive 3 gifts and the others 2 each?

18. If 8 new teachers are to be divided among 4 schools, how many divisions are possible? what if each school must receive 2 teachers?

19. Compute (1) $(3x^2 + y)^5$
   (2) $(x_1 + 2x_2 + 3x_3)^4$

20. Use Stirling's formula to obtain an approximation for 14! and 20!

21. Use Stirling's formula to show that

$$\lim_{n \to \infty} \frac{C(2n, n)\sqrt{\pi n}}{2^{2n}} = 1.$$