in a row. In how many ways can this be done if the people of the same nationality must sit together?

11. Solve problem 10, if they sit at a round table.
12. A class cantains 10 boys and 5 girls.
    i. In how many ways can a teacher choose a committee of 4?
    ii. How many of these committees will contain 2 girls?
    iii. How many of these committees will contain at least one boy?
13. A multiple-choice test consists of 6 questions, each permitting a choice of 4 altrenatives. In how many differen ways can a student check off his answer to these questions?
14. On each business trip, a salesman visits 5 of the major cities in his territory. In how many different ways can he schedule his route (that is the cities and their order) for such a trip?
15. A woman has 8 friends, of whom she will invite 5 to a tea party. How many chioces does she have if 2 of the friends are feuding and will not attend together? How many chioces does she have if 2 of them will only attend together?
16. The game of bridge is played by 4 players, each of whom is dealt 13 cards. How many bridge hands are possible?
17. In how many ways can a man divide 7 gifts among his 3 children if the eldest is to receive 3 gifts and the others 2 each?
18. If 8 new teachers are to be divided among 4 schools, how many divisions are possible? what if each school must receive 2 teachers?
19. Compute (1) $(3x^2 + y)^5$
    (2) $(x_1 + 2x_2 + 3x_3)^4$
20. Use Stirling's formula to obtain an approximation for 14! and 20!
21. Use Stirling's formula to show that

$$\lim_{n \to \infty} \frac{C(2n, n)\sqrt{\pi n}}{2^{2n}} = 1.$$

3

Chapter Three
Probability

Chapter Three- Probability

3.1 Introduction

One of the fundamental tools of statistics is Probability, which had its formal beginning with games of chance in the seventeenth century.

Games of chance, as the name implies, include such actions as spinning a roulette wheel, throwing a die, tossing a coin, drawing a playing card, etc., in which the outcome of a trial is uncertain.

Probability is the study of random or nondeterministic experiments (that is, we wish to consider some experiment results and the repetition of the experiment dose not always produce the same results). For example, suppose one tossed a fair coin repeatedly, keeping a record of the number of heads h in the first n tosses (n=1,2,3 ...), then it has been empirically observed that the ratio $f=h/n$, called the relative frequency becomes stable in the long run, i.e. approaches a limit. This stability is the basis of probability theory.

Any important motivation for studying any branch of probability theory arises from the applications of the theory to practical situations involving random phenomena (including natural and physical phenomena). Many such phenomena can be considered as *random experiments*, the possible outcomes of which are known but the actual results of which cannot be predicted with certainty in advance.

3.2 Random Experiment, Sample Space and Events

Definition 3.2.1

A *random experiment* is a procedure which results in some nondeterministic outcomes in a particular situation.

An *outcome* is a single realization of a phenomenon under consideration. The outcomes need not always be numbers or quantities which are representable in term of numbers.

The set of all possible outcomes of a random experiment is calld the *sample space,* denoted by S or Ω .

When we perform an experiment we will be interested in certain sets of outcomes, that is, subsets of S . Any subset E of the sample space S is called an *event.*

An *elementary event* is an event consisting of only one element of the sample space S . These points are illustrated with some examples.

1. If the experiment consists of tossing a coin once, then S = {H,T},
   where H means that the outcome of the toss is a head and T that it is a tail.
   If E= {H}, then E is the event that head appears.

2. If a couple is planning to have three childern, then S consists of 8 elements, i.e.
   S = {BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}
   where B means that the outcome of the birth is a boy and G that is a girl.
   If E is the event that the couple will have 2 boys, then

   E= {BBG, BGB, GBB}

Also, if F is the event that the couple will have at most one boy, then.

F= {BGG, GBG, GGB, GGG}.

3. If the experiment consists of measuring the lifetime of an electric bulb, then the sample space consists of all nonnegative real numbers, i.e.

$S= \{x : 0 < x < \infty\} = [0,\infty)$

If A =[0,10), then A is the event that the bulb lasts less than 10 hours.

4. If the experiment consists of tossing a pair of dice, then the sample space consists of 36 elements, i.e.

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Define the following events as

A = [the sum on the two dice is 6]
B = [both dice show the same number]
C = [at least one of them is 4].

Then

A = {(1,5) , (2,4), (3,3), (4,2) , (5,1)}
B = {(1,1) , (2,2) , (3,3), (4,4), (5,5), (6,6)}

and

C = {(1,4), (2,4), (3,4) , (4,4), (5,4), (6,4), (4,1), (4,2), (4,3) , (4,5), (4,6)}.

For any two events E and F of the sample space S, We define the following new events:

1. EUF is the event that either E or F (or both) occurs
2. $E \cap F=EF$ is the event that both E and F occur.
3. E' is the event that E does not occur.
4. $E' F$ is the event that only E occurs.

In example (4), we define

$A \cup B$ as the event that the sum on the two dice is 6 or the two dice show the same number, then

A B = {(1,5) , (2,4) , (3,3) , (4,2), (5,1), (1,1), (2,2), (4,4) , (5,5) , (6,6)}.

and

$A \cap B$ is the event that the sum on the two dice is 6 and the two dice show the same number, then

$A \cap B = \{(3,3)\}$.

Example 3.2.1.

Consider the experiment involving the toss of a single die, then

$$S = \{1, 2, 3, 4, 5, 6\}$$

where the outcome i means that i appeared on the die i = 1, 2, 3, 4, 5, 6.

Let

$A = \{i \text{ is even}\}$ and $B = \{i \geqslant 3\}$.

Then $A = \{2,4,6\}$, $B = \{3,4,5,6\}$ and

$A \cup B = \{i \text{ is even or } i \geqslant 3\} = \{2,3,4,5,6\}$,

$A \cap B = \{i \text{ is even and } i \geqslant 3\} = \{4,6\}$.

$A' = \{i \text{ is not even}\} = \{1,3,5\} = \{i \text{ is odd}\}$,

$A/B = \{i \text{ is even but less than } 3\} = \{2\}$,

and $B/A = \{i \text{ is} \geq 3 \text{ but } i \text{ is odd}\} = \{3,5\}$.

In our definition of event as a subset of S (including ϕ and S) ϕ is called the *impossible event* and S is the *certain event*.

Two events E and F of the sample space S are said to be *mutually exclusive* if $E \cap F = \phi$.

In general, the events A_1, A_2 , ... A_n are said to be mutually exclusive if no two of the events have a point in common, i.e.