

Biostatistics – Spring 2026

Lecture 08: Standard Life Tables

Dr. Zaid T. Al-Khaledi
Department of Statistics and Informatics
University of Mosul

April 1, 2026

Introduction

After introducing survival analysis, we now study one of its oldest tools: the **life table**.

A standard life table summarizes survival and mortality patterns across age intervals in a population.

It is widely used in:

- demography,
- public health,
- epidemiology,
- actuarial science,
- health planning.

By the end of this lecture, you should be able to:

- define the main columns of a standard life table,
- compute l_x, d_x, q_x, p_x ,
- compute L_x, T_x, e_x ,
- interpret survival probabilities and life expectancy.

1. What Is a Standard Life Table?

A standard life table describes the survival experience of a hypothetical cohort as it moves through successive ages.

Usually, the table starts with a fixed radix such as:

$$l_0 = 10000$$

or any other chosen number.

The main idea is to follow this cohort through age intervals and calculate how many survive and how many die.

2. Main Columns of the Standard Life Table

2.1 Age x

x is the age at the beginning of the interval.

2.2 Number alive l_x

l_x is the number alive at exact age x .

2.3 Number dying d_x

d_x is the number dying between age x and age $x + 1$.

$$d_x = l_x - l_{x+1}$$

or equivalently,

$$l_{x+1} = l_x - d_x.$$

2.4 Probability of death q_x

$$q_x = \frac{d_x}{l_x}$$

This is the probability that a person alive at age x dies before reaching age $x + 1$.

2.5 Probability of survival p_x

$$p_x = 1 - q_x$$

This is the probability of surviving from age x to age $x + 1$.

3. Person-Years and Life Expectancy

3.1 Person-years lived in the interval L_x

A simple approximation is:

$$L_x = \frac{l_x + l_{x+1}}{2}$$

This represents the average number alive during the interval.

3.2 Total future person-years T_x

$$T_x = L_x + L_{x+1} + L_{x+2} + \dots$$

This is the total future lifetime remaining for all persons alive at age x .

3.3 Life expectancy e_x

$$e_x = \frac{T_x}{l_x}$$

This is the average remaining lifetime for a person aged x .

4. Worked Example

Suppose we start with 10,000 newborns and the numbers dying in the first five years are:

x	d_x
0	800
1	600
2	400
3	300
4	200

Step 1: Compute l_x

Take

$$l_0 = 10000.$$

Then:

$$l_1 = 10000 - 800 = 9200$$

$$l_2 = 9200 - 600 = 8600$$

$$l_3 = 8600 - 400 = 8200$$

$$l_4 = 8200 - 300 = 7900$$

$$l_5 = 7900 - 200 = 7700.$$

Step 2: Compute q_x and p_x

$$q_0 = \frac{800}{10000} = 0.08, \quad p_0 = 0.92$$

$$q_1 = \frac{600}{9200} = 0.0652, \quad p_1 = 0.9348$$

$$q_2 = \frac{400}{8600} = 0.0465, \quad p_2 = 0.9535$$

$$q_3 = \frac{300}{8200} = 0.0366, \quad p_3 = 0.9634$$

$$q_4 = \frac{200}{7900} = 0.0253, \quad p_4 = 0.9747.$$

Step 3: Compute L_x

$$L_0 = \frac{10000 + 9200}{2} = 9600$$

$$L_1 = \frac{9200 + 8600}{2} = 8900$$

$$L_2 = \frac{8600 + 8200}{2} = 8400$$

$$L_3 = \frac{8200 + 7900}{2} = 8050$$

$$L_4 = \frac{7900 + 7700}{2} = 7800.$$

Step 4: Compute T_x

$$T_4 = 7800$$

$$T_3 = 8050 + 7800 = 15850$$

$$T_2 = 8400 + 8050 + 7800 = 24250$$

$$T_1 = 8900 + 8400 + 8050 + 7800 = 33150$$

$$T_0 = 9600 + 8900 + 8400 + 8050 + 7800 = 42750.$$

Step 5: Compute e_x

$$e_0 = \frac{42750}{10000} = 4.275 \approx 4.28$$

$$e_1 = \frac{33150}{9200} \approx 3.60$$

$$e_2 = \frac{24250}{8600} \approx 2.82$$

$$e_3 = \frac{15850}{8200} \approx 1.93$$

$$e_4 = \frac{7800}{7900} \approx 0.99.$$

5. Interpretation of Results

- At age 0, the expected remaining lifetime is about 4.28 years.
- At age 2, the expected remaining lifetime is about 2.82 years.
- As age increases, the remaining life expectancy decreases.

This is a very important point:

Life expectancy at age x means the expected future years of life for someone who has already survived to age x .

6. Common Student Mistakes

- Confusing d_x with q_x .
- Forgetting that $p_x = 1 - q_x$.
- Computing L_x incorrectly.
- Computing T_x from top to bottom instead of bottom to top.
- Interpreting e_x as total age instead of remaining expected life.

7. Summary

- Standard life tables summarize survival by age intervals.
- The main columns are $l_x, d_x, q_x, p_x, L_x, T_x, e_x$.
- q_x is probability of death and p_x is probability of survival.
- e_x is the remaining life expectancy at age x .

Homework (HW)

HW1

Suppose $l_0 = 5000$ and the deaths are:

x	d_x
0	300
1	250
2	200
3	150

Compute:

1. l_x
2. q_x
3. p_x

HW2

For the data in HW1, compute:

1. L_x
2. T_x
3. e_x

HW3

Explain in words the meaning of:

$$q_2 = 0.05, \quad p_2 = 0.95, \quad e_0 = 3.8.$$