Solution

Step 1 State the hypothesis and identify the claim. Since we are interested to see if there has been an increase in deposits, the deposits 3 years ago must be less than the deposits today; hence, the differences must be significantly less 3 years ago than they are today. Hence the mean of the differences must be less than zero.

H_0: $\mu_D = 0$ and H_1: $\mu_D < 0$ (claim)

Step 2 Find the critical value. The degrees of freedom are $n - 1$, or $9 - 1 = 8$. The critical value for a left-tailed test with $\alpha = 0.05$ is -1.860.

Step 3 Compute the test value.

a. Make a table.

3 years ago (X_1)	Now (X_2)	A $D = X_1 - X_2$	B $D^2 = (X_1 - X_2)^2$
11.42	16.69		
8.41	9.44		
3.98	6.53		
7.37	5.58		
2.28	2.92		
1.10	1.88		
1.00	1.78		
0.90	1.50		
1.35	1.22		

b. Find the differences and place the results in column A.

$11.42 - 16.69 = -5.27$
$8.41 - 9.44 = -1.03$
$3.98 - 6.53 = -2.55$
$7.37 - 5.58 = +1.79$
$2.28 - 2.92 = -0.64$
$1.10 - 1.88 = -0.78$
$1.00 - 1.78 = -0.78$
$0.9 - 1.50 = -0.60$
$1.35 - 1.22 = +0.13$
$\Sigma D = 9.73$

c. Find the means of the differences.

$$\overline{D} = \frac{\Sigma D}{n} = \frac{-9.73}{9} = -1.081$$

d. Square the differences and place the results in Column B.

$(-5.27)^2 = 27.7729$
$(-1.03)^2 = 1.0609$
$(-2.55)^2 = 6.5025$
$(+1.79)^2 = 3.2041$
$(-0.64)^2 = 0.4096$
$(-0.78)^2 = 0.6084$
$(-0.78)^2 = 0.6084$
$(-0.60)^2 = 0.3600$
$(+0.13)^2 = 0.1690$
$\Sigma D^2 = 40.5437$

The formulas for this t test are summarized next.

Formulas for the t Test for Dependent Samples

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}}$$

with d.f. $= n - 1$ and where

$$\overline{D} = \frac{\Sigma D}{n} \quad \text{and} \quad s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

Example 9–7

Cholesterol Levels

A dietitian wishes to see if a person's cholesterol level will change if the diet is supplemented by a certain mineral. Six subjects were pretested, and then they took the mineral supplement for a 6-week period. The results are shown in the table. (Cholesterol level is measured in milligrams per deciliter.) Can it be concluded that the cholesterol level has been changed at $\alpha = 0.10$? Assume the variable is approximately normally distributed.

Subject	1	2	3	4	5	6
Before (X_1)	210	235	208	190	172	244
After (X_2)	190	170	210	188	173	228

Solution

Step 1 State the hypotheses and identify the claim. If the diet is effective, the before cholesterol levels should be different from the after levels.

H_0: $\mu_D = 0$ and H_1: $\mu_D \neq 0$ (claim)

Step 2 Find the critical value. The degrees of freedom are 5. At $\alpha = 0.10$, the critical values are ± 2.015.

Step 3 Compute the test value.

a. Make a table.

Before (X_1)	**After (X_2)**	**A** $D = X_1 - X_2$	**B** $D^2 = (X_1 - X_2)^2$
210	190		
235	170		
208	210		
190	188		
172	173		
244	228		

b. Find the differences and place the results in column A.

$$\begin{aligned} 210 - 190 &= 20 \\ 235 - 170 &= 65 \\ 208 - 210 &= -2 \\ 190 - 188 &= 2 \\ 172 - 173 &= -1 \\ 244 - 228 &= 16 \\ \Sigma D &= 100 \end{aligned}$$

Step 5 Summarize the results. There is not enough evidence to support the claim that the mineral changes a person's cholesterol level.

The steps for this t test are summarized in the Procedure Table.

Procedure Table

Testing the Difference Between Means for Dependent Samples

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s).

Step 3 Compute the test value.

a. Make a table, as shown.

		A	B
X_1	X_2	$D = X_1 - X_2$	$D^2 = (X_1 - X_2)^2$
⋮	⋮		
		$\Sigma D =$ ______	$\Sigma D^2 =$ ______

b. Find the differences and place the results in column A.

$$D = X_1 - X_2$$

c. Find the mean of the differences.

$$\overline{D} = \frac{\Sigma D}{n}$$

d. Square the differences and place the results in column B. Complete the table.

$$D^2 = (X_1 - X_2)^2$$

e. Find the standard deviation of the differences.

$$s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

f. Find the test value.

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}} \quad \text{with d.f.} = n - 1$$

Step 4 Make the decision.

Step 5 Summarize the results.

Unusual Stat

About 4% of Americans spend at least one night in jail each year.

The P-values for the t test are found in Table F. For a two-tailed test with d.f. $= 5$ and $t = 1.610$, the P-value is found between 1.476 and 2.015; hence, $0.10 < P\text{-value} < 0.20$. Thus, the null hypothesis cannot be rejected at $\alpha = 0.10$.

If a specific difference is hypothesized, this formula should be used

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}}$$

where μ_D is the hypothesized difference.