Lecture 3

Framework and Notation

- In all the scheduling problems considered, the number of jobs and the number of machines are assumed to be finite.
- The number of jobs is denoted by n, and the number of machines by m.
- Usually, the subscript i refers to a job while the subscript j refers to a machine.
- If a job requires several operations (processing steps), i.e. a job J_i consists of a number n_i of operations $O_{i1}, \cdots, O_{i,n_i}$ then we will have a data for each operation.
- The following pieces of data are associated with job i:
  - Processing time (p_{ij}): The processing time p_{ij} represents the time job i has to spend on machine j. The subscript j is omitted if the processing time of job i does not depend on the machine or if it only needs processing on one machine.
  - Release date (r_i): The release date r_i of job i is also known as the ready date. It is the time the job arrives at the system, i.e., the earliest time at which job i can start its processing.
  - Due date (d_i): The due date d_i of job i represents the committed shipping or completion date (i.e., the date the job is promised to the customer). Completion of a job after its due date is allowed, but then a penalty is incurred. When a due date must be met it is referred to as a deadline and denoted by $\bar{d}_i$.
  - Weight (w_i): The weight w_i of job i is a priority factor, reflecting the importance of job i relative to other jobs in the system.

  The four pieces of data listed above are static data, since they do not depend on the schedule. Conversely, data that are not fixed in advance and that depend on the schedule are referred to as dynamic data. The most important dynamic data are the following:

  - Starting time S_{ij}: The starting time S_{ij} is the time when job i starts its processing on machine j. If the subscript j is omitted, then S_i refers to the time when job i starts with its first processing in the system.
  - Completion time C_{ij}: The completion time C_{ij} is the time when job i is completed on machine j. If the subscript j is omitted, then C_i refers to the time when job i leaves the system.
  - Flowtime F_i: The time job i spends in the system, $F_i = C_i - r_i$.

Performance Measures and Objectives

Several types of scheduling objectives are important. In practice, the overall objective is often a composite of several basic objectives. The most important of these basic objectives is described below.

- Makespan Objective: The makespan is important when there is a finite number of jobs. The makespan is denoted by C_{max} and is defined as the time when the last job leaves the system, i.e.,

$$C_{max} = \max\{C_1, \cdots, C_n\}$$

  where C_i is the completion time of job i.

- Due Date Related Objectives: Several important objectives are related to due dates. First, the scheduler is often concerned with minimizing the maximum lateness. Job lateness is defined as follows. Let d_i denote the due date of job i. The lateness of job i is then

$$L_i = C_i - d_i$$

  The maximum lateness is defined as

$$L_{max} = \max\{L_1, \cdots, L_n\}$$

  Minimizing the maximum lateness is, in a sense, equivalent to minimizing the worst performance of the scheduling.
  Another important due date related objective is the number of tardy jobs. This objective does not focus on how tardy a job actually is, but only on whether or not it is tardy. A due date related criterion that addresses this concern is the total tardiness or, equivalently, the average tardiness. The tardiness of job i is defined as

$$T_i = \max(C_i - d_i, 0)$$

  and the objective function is

$$\sum_{i=1}^{n} T_i$$

  Suppose different jobs carry different priority weights, where the weight of job i is w_i. The larger the weight of the job, the more important it is. Then, a more general version of the objective function is the total weighted tardiness

$$\sum_{i=1}^{n} w_i T_i$$

- Flowtime Related Objectives: Several important objectives are related to flowtime like total flowtime

$$\sum_{i=1}^{n} F_i$$

  and maximum flowtime

$$F_{max} = max\{F_1, \cdots, F_n\}$$