

Exercise Tautology

Example1: Construct a truth table to show that $p \vee \neg p$ is a tautology

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Since all entries under $p \vee \neg p$ are T , the formula is a tautology

Example2: Construct a truth table to show that $(p \wedge q) \rightarrow q$ is a tautology

p	q	$p \wedge q$	$(p \wedge q) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Since all entries under $(p \wedge q) \rightarrow q$ are T , the formula is a tautology

Example3: Construct a truth table to show that $p \rightarrow (p \vee q)$ is a tautology

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Since all entries under $p \rightarrow (p \vee q)$ are T , the formula is a tautology

Example4: Construct a truth table to show that $p \wedge q \leftrightarrow (q \wedge p)$ is a tautology

p	q	$p \wedge q$	$q \wedge p$	$p \wedge q \leftrightarrow (q \wedge p)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	F	T

Since all entries under $p \wedge q \leftrightarrow (q \wedge p)$ are T , the formula is a tautology

Example5: Construct a truth table to show that $(p \rightarrow q) \vee (q \rightarrow p)$ is a tautology

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \vee (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Since all entries under $(p \rightarrow q) \vee (q \rightarrow p)$ are T , the formula is a tautology

Exercise on Predicates

1) Let $p(x)$ denote the statement " **x is an even number**". What are the truth values of $P(8)$ and $P(9)$

Answer:

- $P(8)$: True
- $P(9)$: False

2) Let $p(x)$ denote the statement " **$x \leq 4$** ". What are the truth values of $P(4)$ and $P(7)$

Answer:

- $P(4)$: True
- $P(7)$: False

3) Let $p(x)$ denote the statement " **$x + y = 10$** ". What are the truth values of $Q(6,4)$ and $Q(3,5)$

Answer:

- $P(6,4)$: True
- $P(3,5)$: False

4) Let $p(x)$ denote the statement "**The word x contains the letter (a)**". What are the truth values of $P(\text{apple})$ and $P(\text{orange})$

Answer:

- $P(\text{apple})$: True
- $P(\text{orange})$: True

5) Let $p(x)$ denote the statement " **$x \leq 10$** ". What are the truth values of $P(10)$ and $P(14)$

Answer:

- $P(10)$: True
- $P(14)$: False

Exercise on Quantification

1) Let $p(x)$ denote the statement " **x is positive**". What is the truth values of $\forall x p(x)$ over all integers

Solution: Not all integers are positive (for example, -1) . Thus, $\forall x p(x)$ is false

2) Let $p(x)$ denote the statement " **$x^2 \geq 0$** ". What is the truth values of $\forall x p(x)$ over all real numbers

Solution: Since For **any real number** x , squaring it gives a **non-negative** result:

- If $x > 0$, then $x^2 > 0$
- If $x = 0$, then $x^2 = 0$
- If $x < 0$, then $x^2 > 0$

Therefore, $x^2 \geq 0$ holds for **all real numbers**.

3) Let $Q(x)$ denote the statement " **x is even** ". What is the truth values of $\exists x Q(x)$ over all integer numbers

Solution: There exists an even integer (for example ,2). Thus , $\exists x Q(x)$ is true

4) Let $P(x)$ denote the statement " **$x > 10$** ". What is the truth values of $\exists x P(x)$ in the domain $\{1,2,3,4,5\}$

Solution: No element in the domain is greater than 10 . Thus , $\exists x P(x)$ is false

5) Let $p(x)$ denote the statement " **$x^2 = 9$** ". What is the truth values of $\exists! x p(x)$ over all real numbers

Solution: Since the solution is not unique, $\exists! x p(x)$ is false.