

Lecture 5 Exercises – Discrete Structure

1. Composition of Functions

1) If $f(x)=x+2$ and $g(x)=3x$, find $(g \circ f)(x)$.

Solution: $g(f(x)) = g(x+2) = 3(x+2) = 3x + 6$

2) Let $f(x)=2x$ and $g(x)=x^2$. Find $(f \circ g)(2)$.

Solution: $f(g(2)) = 2x^2 = 2(4) = 8$

3) If $f(x)=x-1$ and $g(x)=x+5$, find $(g \circ f)(x)$.

Solution: $g(f(x)) = g(x-1) = x + 4$

4) If $f(x)=3x$ and $g(x)=x-2$, find $(f \circ g)(1)$.

Solution: $f(g(1)) = 3(x-2) = 3x-6 = -3$

5) If $f(x)=x^2$ and $g(x)=2x$, is $(f \circ g) = (g \circ f)$?

Solution: No, because $f(g(x))=4x^2$ while $g(f(x))=2x^2$

2. Inverse Functions

1) Find the inverse of $f(x)=x+4$.

Solution: Let's work through this step by step:

Function given:

$$[f(x) = x + 4]$$

Steps to find the inverse:

Replace $(f(x))$ with (y) : $[y = x + 4]$

Swap (x) and (y) : $[x = y + 4]$

Solve for (y) : $[y = x - 4]$

Final Answer:

$$[f^{-1}(x) = x - 4]$$

2) Does the function $f(x)=2x$ have an inverse? Find it.

Solution: Yes, the function ($f(x) = 2x$) does have an inverse

Step 1: Check if it has an inverse

A function has an inverse if it is **one-to-one** (each input maps to a unique output).

($f(x) = 2x$) is a linear function with slope (2). Linear functions with nonzero slope are always one-to-one.

So, **yes, it has an inverse.**

Step 2: Find the inverse

Write the function as: $[y = 2x]$

Swap (x) and (y): $[x = 2y]$

Solve for (y): $\left[y = \frac{x}{2} \right]$

Final Answer:

$$f^{-1}(x) = x/2$$

3) Find $f^{-1}(x)$ if $f(x) = 3x - 6$.

$$\text{Solution: } f^{-1}(x) = (x+6)/3$$

4) Show that $f(x) = x^3$ has an inverse.

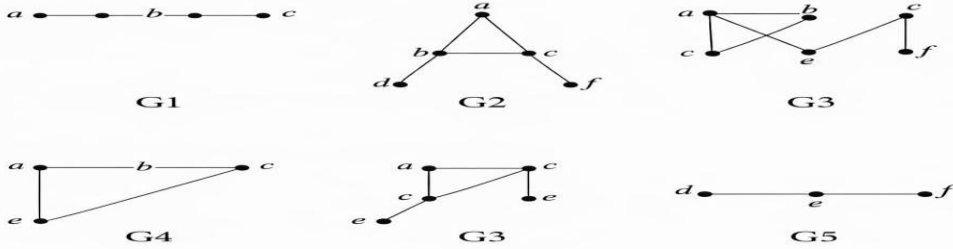
$$\text{Solution: Yes, } f^{-1}(x) = \sqrt[3]{x}$$

5) Does $f(x) = x^2$ have an inverse?

Solution: No, it is not one-to-one

3. Trees

Which diagrams represent a tree?



- Just the first and the fifth shape

Based on the principles of Graph Theory, a Tree is a graph that satisfies two fundamental conditions:

- * Connected: There is a path between every pair of vertices (nodes).
- * Acyclic: It contains no cycles (no closed loops where you can return to the starting point without backtracking).

Analysis of the Graphs:

Graphs that are Trees:

- * G1: This is a simple linear path connecting four points. It is fully connected and has no cycles. (Tree)
- * G5: This is a path connecting points d, e, and f. It is connected and has no cycles. (Tree)

Graphs that are NOT Trees:

- * G2: Contains a cycle (a triangle) between points a, b, and c.
- * G3 (Top right): Contains multiple closed cycles (e.g., the square-like path between a, b, e, and c).
- * G4: Contains a clear cycle forming a triangle between points a, c, and e.

* G3 (Bottom middle): Contains a closed loop (cycle) in its upper section.

Summary:

The graphs that qualify as trees are G1 and G5.

m-ary tree full m-ary tree

