

Examples

1. Restricted Quantifiers

Choose the correct answer from the following statements:

1. $\forall x > 0 (x^2 > 0)$ over $\mathbb{R}$ - True
2. $\forall x < 0 (x^2 > 0)$ over $\mathbb{R}$ - True
3. $\exists x \geq 0 (x^2 = 4)$ - True
4. $\forall x \neq 0 (1/x \neq 0)$ - True
5. $\exists x < 0 (x^3 > 0)$ - False

2. Nested Quantifiers

Choose the correct answer from the following statements:

1. $\forall x \forall y (x + y = y + x)$ - True
2. $\forall x \exists y (x + y = 0)$ - True
3. $\exists x \forall y (x + y = 0)$ - False
4. $\exists x \exists y (x^2 + y^2 = 1)$ - True
5. $x \exists y (x < y)$ - True

3. Functions

Choose the correct answer from the following statements:

1. $f(x) = x + 1$ from $\mathbb{R}$ to $\mathbb{R}$ - True
2. $f(x) = x^2$ from $\mathbb{R}$ to $\mathbb{R}$ - True
3. $f(x) = 2x + 3$ from $\mathbb{Z}$ to $\mathbb{Z}$ - True
4. $A = \{1,2,3\}, B = \{10,20,30,40\}$
 $f(1) = 10, f(2) = 20$ - false

4. One-to-One (Injective)

When we have the function from $\mathbb{R}$ to $\mathbb{R}$

1. $f(x) = 2x$ - True
2. $f(x) = x + 5$ - True
3. $f(x) = x^3$ - True
4. $f(x) = -x$ - True
5. $f(x) = 3x - 1$ - True
6. $f(x) = x^2$

Domain: $\mathbb{R}$ (all real numbers)

Codomain: $\mathbb{R}$

Reason: If we take $x_1 = 2$ and $x_2 = -2$, then

$$f(2) = 4, f(-2) = 4$$

Here $f(x_1) = f(x_2)$ but $x_1 \neq x_2$. Therefore, $f(x) = x^2$ is not injective on $\mathbb{R}$.

7. $f(x) = |x|$

Domain: $\mathbb{R}$ (all real numbers)

Codomain: $[0, \infty)$

Reason: If we take $x_1 = 3$ and $x_2 = -3$, then

$$f(3) = 3, f(-3) = 3$$

Again, $f(x_1) = f(x_2)$ but $x_1 \neq x_2$. Therefore, $f(x) = |x|$ is not injective on $\mathbb{R}$.

5. Onto (Surjective)

1. $f(x) = 2x$ from $\mathbb{Z}$ to the even integers

Domain: integers $(\{-3, -2, -1, 0, 1, 2, 3\})$

Codomain: even integers $(\{-6, -4, -2, 0, 2, 4, 6\})$

Example: $(f(2) = 4)$. Every even integer appears in the codomain.

2. $(f(x) = |x|)$ from $\mathbb{R}$ to $\mathbb{R}^+$

Domain: real numbers $(\{-3, -2, -1, 0, 1, 2, 3\})$

Codomain: non – negative real numbers $\{0, 1, 2, 3\}$

Example: $(f(-2) = 2)$. Every non – negative number appears in the codomain.

3. $f(x) = \lfloor x \rfloor$ from $\mathbb{R}$ to $\mathbb{Z}$

Domain: real numbers $(\{-2.7, -1.3, 0.5, 1.9, 2.1\})$

Codomain: integers $(\{-3, -1, 0, 1, 2\})$

Example: $(f(1.9) = 1)$. Every integer appears in the codomain.

4. $f(x) = x - 5$ **from $\mathbb{Z}$ to $\mathbb{Z}$**

Domain: integers $\mathbb{Z}$

Codomain: integers $\mathbb{Z}$

Example: $(f(5) = 0)$. Every integer in the codomain has a preimage.

5. $f(x) = x^2$ from $\mathbb{R}$ to $\mathbb{R}$

Domain: all real numbers ($\{-2, -1, 0, 1, 2\}$)

Codomain: all real numbers ($\{-2, -1, 0, 1, 2, 3, \dots\}$)

Problem: Negative numbers like (-1) or (-5) never appear in the codomain.

Therefore, this function is **not onto**.

6. $f(x) = \sin x$ from $\mathbb{R}$ to $\mathbb{R}$

Domain: real numbers $\{0, \pi/2, \pi, 3\pi/2\}$

Codomain: all real numbers $\{-2, -1, 0, 1, 2, 3, \dots\}$

Example: $f(0) = 0$, $f(\pi/2) = 1$

Problem: Values outside the interval $([-1, 1])$, like (2) or (-3) , never appear.

Therefore, this function is **not onto**.

6. bijection.

1. $(f(x) = x + 1)$ from $\mathbb{Z}$ to $\mathbb{Z}$

- Injective (one – to – one): Yes, because if $(x_1 + 1 = x_2 + 1)$, then $(x_1 = x_2)$.
 - Surjective (onto): Yes, for any integer (y) , we can take $(x = y - 1)$ so that $(f(x) = y)$.
- This is a bijection.

2. $(f(x) = 2x)$ from $(\mathbb{R})$ to $(\mathbb{R})$

- *Injective: Yes, if $(2x_1 = 2x_2)$, then $(x_1 = x_2)$.*
 - *Surjective: Yes, for any real number (y) , we can take $(x = y/2)$.*
- This is a bijection.*

3. $(f(x) = x^3)$ from $(\mathbb{R})$ to $(\mathbb{R})$

- *Injective: Yes, because (x^3) is strictly monotonic on $(\mathbb{R})$.*
 - *Surjective: Yes, for any real number (y) , we can take $(x = \sqrt[3]{y})$.*
- This is a bijection.*

4. $(f(x) = x)$ from (A) to (A)

- *This is the identity function.*
 - *Always injective and surjective.*
- This is a bijection.*

5. $(f(x) = x - 3)$ from (Z) to (Z)

- *Injective: Yes, if $(x_1 - 3 = x_2 - 3)$, then $(x_1 = x_2)$.*
 - *Surjective: Yes, for any integer (y) , we can take $(x = y + 3)$.
This is a bijection.*
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