LECTURES ON THE ELECTRICITY AND MAGNETISM

محاضرات حول الكهربائية والمغناطيسية

For Second Year Physics
Education College of Pure Science
Mosul University

للسنة الثانية، قسم الفيزياء

كلية التربية للعلوم الصرفة

جامعة الموصل

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Chapter One

Magnetic Field of the Electric Current

1.1 Magnetic Fields

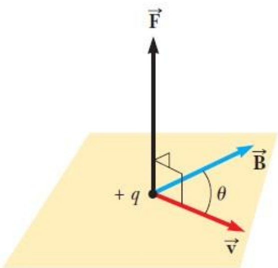
Experiments show that a *stationary (motionless) charged particle* doesn't interact with a *static magnetic field*. But, when a charged particle is *moving* through a magnetic field, *a magnetic force* acts on it. This *force* has its *maximum value* when the *charge* moves in a direction perpendicular to the magnetic field lines. Magnetic force decreases in value at other angles (<90°), and becomes zero when the particle moves along the magnetic field lines.

This is quite different from the electric force, which exerts a force on a charged particle whether it's moving or at rest. Further, the electric force is directed parallel to the electric field while the magnetic force on a moving charge is directed perpendicular to the magnetic field.

We can describe the properties of the magnetic field $(\vec{B})$ at a certain point in terms of the magnetic force exerted on a test charge at that point. Our test object is a charge (q) moving with velocity $(\vec{v})$. It is found experimentally that the strength of the magnetic force on the particle is directly proportional to the magnitude of the charge (q), the magnitude of the velocity $(\vec{v})$, the strength of the external magnetic field $(\vec{B})$, and the sine of the angle (θ) between the direction of $(\vec{v})$ and the direction of $(\vec{B})$ **(Figure 1.1)**. These observations can be summarized by writing the magnitude of the magnetic force $(\vec{F})$ as [1]:

$$F = qvB\sin\theta \qquad (1.1)$$

Figure 1.1: The direction of the magnetic force on a positively charged particle moving with a velocity $(\vec{v})$ in the presence of a magnetic field $(\vec{B})$.



This expression (1.1) is used to define the magnitude of the magnetic field $(\vec{B})$ as:

$$B = \frac{F}{qv\sin\theta} \qquad (1.2)$$

If $(\vec{F})$ is in newton's (symbol: N), (q) in coulombs (symbol: C), and $(\vec{v})$ in meters per second (m/s), then the International System of Units (SI) of magnetic field $(\vec{B})$ is the tesla (T), also called the weber (Wb) per square meter (1T = 1 Wb/m^2).

If a (1C) charge moves in a direction perpendicular to a magnetic field of magnitude (1T) with a speed of (1 m/s), the magnetic force exerted on the charge is (1N). We can express the units of ($\vec{B}$) as:

$$B = T = \frac{Wb}{m^2} = \frac{N}{C.m/s} = \frac{N}{A.m} \qquad (1.3);$$ Where C/s ≡ Ampere (I=Q/t).

In practice, the cgs (centimeter-gram-second) unit for magnetic field, the gauss (G), is often used. The gauss is related to the tesla through the conversion:

$$1\,T = 10^4\,G$$

Conventional laboratory magnets can produce magnetic fields as large as about 25000 G, or 2.5 T. Superconducting magnets that can generate magnetic fields as great as $3 \times 10^5\,G$, 30 T, have been constructed. These values can be compared with the value of Earth's magnetic field near its surface, which is about 0.5 G, or $0.5 \times 10^{-4}\,T$.

From equation (1.1) we see that the force on a charged particle *moving* in a magnetic field has its maximum value when the particles motion is *perpendicular* to the magnetic field, corresponding to ($\theta = 90^{\circ}$, so that $\sin\theta = 1$ **(Figure 1.2)**. The magnitude of this maximum force has the value:

$$F_{max} = qvB \qquad (1.4)$$

Also from equation (1.1), $\vec{F}$ is zero when ($\vec{v}$) is parallel to ($\vec{B}$), corresponding to ($\theta = 0°\ or\ 180°$), so no magnetic force is exerted on a charged particle when it moves in the direction of the magnetic field or opposite the field.

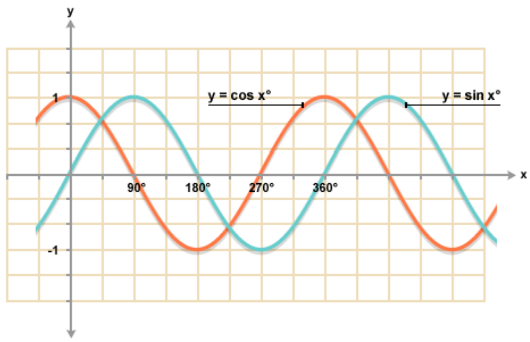


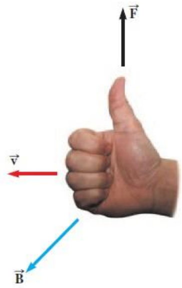
Figure 1.2 Display the sine and cosine waveforms.

Experiments show that the direction of the magnetic force is *always* perpendicular to both ($\vec{v}$) and ($\vec{B}$), as shown in **Figure 1.1**, for a positively charged particle.

To determine the direction of the force, we employ (use) the *right-hand rule number 1* (RHR-1) **(Figure 1.3)**:

1) Point the fingers of your right hand in the direction of the velocity ($\vec{v}$).
2) Curl the fingers in the direction of the magnetic field ($\vec{B}$), moving through the smallest angle (as in **Figure 1.3**).
3) Your thumb is now pointing in the direction of the magnetic force $\vec{F}$ exerted on a positive charge.

Figure 1.3: Right-hand rule number 1 for determining the direction of the magnetic force on a positive charge moving with a velocity in a magnetic field.

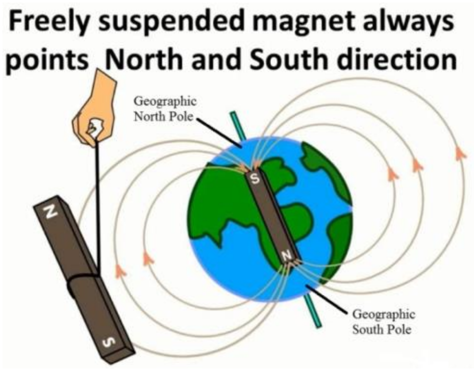


If the charge is negative rather than positive, the force $\vec{F}$ is directed *opposite* that shown in Figures 1.1 and 1.3. So if q is negative, simply use the right-hand rule to find the direction for positive q, and then reverse that direction for the negative charge.

Exercise 1:(HW)

The north-pole end of a bar magnet is held near a stationary positively charged piece of plastic. Is the plastic (a) attracted, (b) repelled, or (c) unaffected by the magnet?

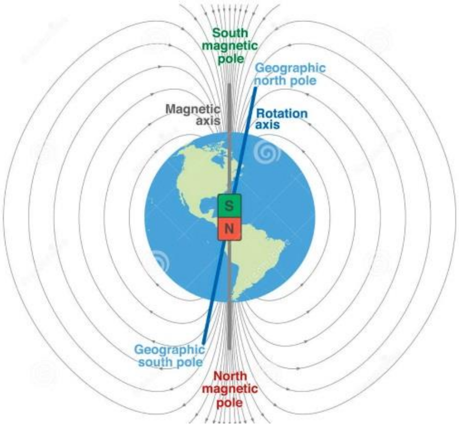
If a bar magnet is suspended **(as shown in the Figure below)** from its midpoint and can swing freely in a horizontal plane, it will rotate until its North Pole points to the Earth's geographic North Pole and its South Pole points to the Earth's geographic South Pole. (The same idea is used in the construction of a simple compass).



Example 1:

A proton moves with a speed of (1×10^5 m/s) through Earth's magnetic field **(Figure 1.1)**, which has a value of (55 μT), at a particular location. When the proton moves eastward, the magnetic force acting on it is directed straight upward, and when it moves northward, no magnetic force acts on it. **(a)** What is the direction of the magnetic field, and **(b)** what is the strength of the magnetic force when the proton moves eastward? Note that the proton charge is 1.6×10^{-19} C.

Figure 1.1: The Earth's Magnetic Field.



(a) Find the direction of the magnetic field?

No magnetic force acts on the proton when it's going north, so the angle that a proton makes with the magnetic field direction must be either 0° or 180°. Therefore, the magnetic field ($\vec{B}$) must point either north or south. Now apply the RHR. When the particle travels east, the magnetic force is directed upward. Point your thumb in the direction of the force and your fingers in the direction of the velocity eastward. When you curl your fingers, they point north, *which must therefore be the direction of the magnetic field*.

(b) Find the magnitude of the magnetic force?

From part (a), the angle between the velocity of the proton and the magnetic field is 90°.

$$F = qvB\sin\theta = (1.6\times10^{-19}C)\left(1\times10^{5}\frac{m}{s}\right)(55\times10^{-6}T)\sin(90°)$$

$$F = 8.8\times10^{-19}N$$

Exercise 2:(HW)

Suppose an electron is moving due west in the same magnetic field as in Example 1 at a speed (2.5×10^5 m/s). Find the (a) magnitude and (b) direction of the magnetic force on the electron?

Answer: **2.2×10^{-18} N, Straight up.**

2.2 Torque On A Current Loop In Uniform Magnetic Field

In the previous (preceding) chapter we showed how a magnetic force ($F = BIl \sin\theta$) is exerted on a current carrying conductor when the conductor is placed in an external magnetic field. With this as a starting point, we now show that a torque is exerted on a current loop placed in a magnetic field. The results of this analysis will be of great practical value when we discuss generators and motors.

Torque is the twisting force that tends to cause rotation, Or "the force or power that makes something turns around a central point". The point where the object rotates is known as the axis of rotation (Pivot axis).

Consider a rectangular loop carrying current (I) in the presence of an external uniform magnetic field in the plane of the loop, as shown in Figure 2.1a.

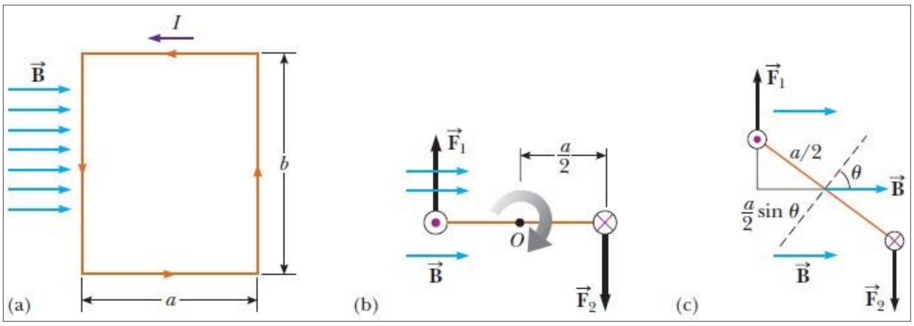
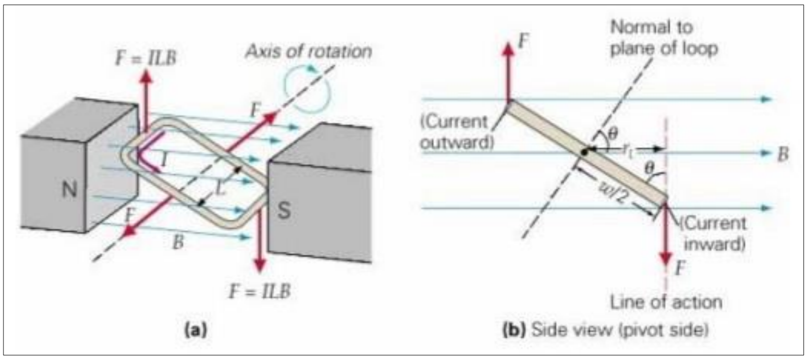


Figure 2.1: **(a)** Top view of a rectangular loop in a uniform magnetic field (**B**). No magnetic forces act on the sides of length (a) parallel to (**B**), but forces do act on the sides of length (b). **(b)** A side view of the rectangular loop shows that the forces (F_1) and (F_2) on the sides of length (b) create a torque that tends to twist the loop clockwise. **(c)** If (**B**) is at an angle (θ) with a line perpendicular to the plane of the loop, the torque is given by ($BIA \sin\theta$).

For a deep (more) understanding see the pictures below:



The forces on the sides of length (a) are zero because these wires are parallel to the field. The magnitudes of the magnetic forces on the sides of length (b) are:

$$F_1 = F_2 = BIb$$

The direction of F_1, the force on the left side of the loop, is out of the page (Figure 2.1a), and that of F_2, the force on the right side of the loop, is into the page (Figure 2.1a). If we look (view) the loop from the side, as in Figure 2.1b, the forces are directed as shown. If we assume that the loop is pivoted (rotated), so that it can rotate about point (O), we see that these two forces produce a torque about (O) that rotates the loop clockwise. The magnitude of this torque, τ_{max}, is:

$$\tau_{max} = F_1 \frac{a}{2} + F_2 \frac{a}{2} = (BIb)\frac{a}{2} + (BIb)\frac{a}{2} = BIab$$

Where the moment arm about (O) is $(a/2)$ for both forces.

Because the area of the loop is $(A = ab)$, the torque can be expressed as:

$$\tau_{max} = BIA \qquad (2.1)$$

This result is valid only when the magnetic field is parallel to the plane of the loop (i.e. (**B**) is at an angle $(\theta = 90°)$ with a line perpendicular to the plane of the loop), as in Figure 2.1b. If the field makes an angle (θ) with a line perpendicular to the plane of the loop, as in Figure 2.1c, the moment arm for each force is given by $(a/2)\ sin\theta$. An analysis similar to the previous gives, for the magnitude of the torque:

$$\tau = BIA\ sin\theta \qquad (2.2)$$

This result shows that the torque has the maximum value (BIA) when the field is parallel to the plane of the loop (i.e. (**B**) is at an angle $(\theta = 90°)$ with a line perpendicular to the plane of the loop).

And the torque is zero when the field is perpendicular to the plane of the loop (i.e. (**B**) is at an angle $(\theta = 0)$ with a line perpendicular to the plane of the loop.

As seen in Figure 2.1c, the loop tends to rotate to smaller values of (θ), (so that the normal to the plane of the loop (i.e line perpendicular to the plane of the loop) rotates toward the direction of the magnetic field).

Although the previous (foregoing) analysis was for a rectangular loop, a more general derivation shows that Equation 2.2 applies regardless of the shape of the loop. Further, the torque on a coil with (N) turns is:

$$\tau = BIAN\ sin\theta \qquad (2.3)$$

The quantity ($\mu = IAN$) is defined as the magnitude of a vector ($\vec{\mu}$) called the magnetic moment of the coil. The vector ($\vec{\mu}$) always points perpendicular to the plane of the loop (s). The angle (θ) in Equations 2.2 and 2.3 lies between the directions of the magnetic moment ($\vec{\mu}$) and the magnetic field (**B**). The magnetic torque can then be written:

$$\tau = \mu B\ sin\theta \qquad (2.4)$$

Example 2.1: The Torque on a Circular Loop in a Magnetic Field

A circular wire loop of radius ($1\ m$) is placed in a magnetic field of magnitude ($0.5\ T$). The normal to the plane of the loop makes an angle of (30°) with the magnetic field (Fig. 1.1a). The current in the loop is ($2\ A$) in the direction shown. **(a)** Find the **(i)** magnetic moment of the loop and the **(ii)** magnitude of the torque at this instant. **(b)** The same current is carried by the rectangular (2 m) by (3 m) coil with three loops shown in Figure 1.1b. Find the **(i)** magnetic moment of the coil and the **(ii)** magnitude of the torque acting on the coil at that instant.

Solution:

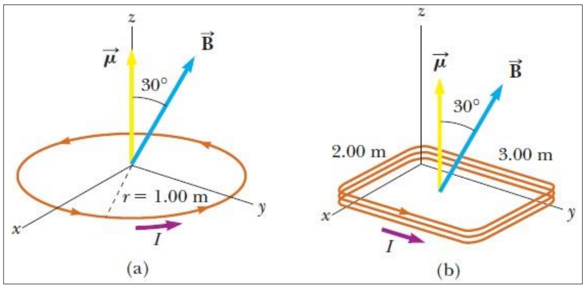


Figure 1.1: **(a)** A circular current loop in an external magnetic field (**B**). **(b)** A rectangular current loop in the same field.

(a) Find the **(i)** magnetic moment (μ) of the circular loop and the **(ii)** magnetic torque (τ) exerted on it.

(i) $\mu = IAN \quad \Rightarrow$

$A = \pi r^2 \quad = \pi\,(1\ m^2) = 3.14\ m^2$

$\mu = IAN \quad = (2\ A)(3.14\ m^2)(1) = 6.28\ A.m^2$

(ii) $\tau = \mu B\ sin\theta = (6.28\ A.m^2)(0.5\ T)(\sin 30°) = 1.57\ N.m$

(b) Find the **(i)** magnetic moment of the rectangular coil and the **(ii)** magnetic torque exerted on it.

$A = L \times H = (2\ m)(3\ m) = 6\ m^2$

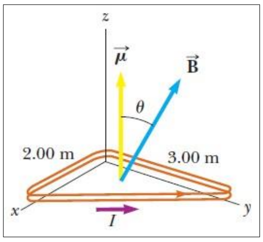
(i) $\mu = IAN \quad = (2\ A)(6\ m^2)(3) = 36\ A.m^2$

(ii) $\tau = \mu B\ sin\theta = (36\ A.m^2)(0.5\ T)(\sin 30°) = 9\ N.m$

Remarks: In calculating a magnetic torque, it's not strictly necessary to calculate the magnetic moment. Instead, Equation 2.3 can be used directly.

Exercise 2.1:(HW)

Suppose a right triangular coil with base of $(2\ m)$ and height $(3\ m)$ having two loops carries a current of $(2\ A)$ as shown in the Figure below. **Find** the magnetic moment and the torque on the coil. The magnetic field is again $(0.5\ T)$ and makes an angle of $(30°)$ with respect to the normal direction.



Answer: $\boldsymbol{\mu = 12\ A.m^2,\ \tau = 3\ N.m}$

2.3 Electric Motors

It's hard to imagine life in the 21st century without electric motors. Some devices (appliances) that contain motors include computer disk drives, CD players, VCR and DVD players, food processors and blenders, car starters, furnaces, and air conditioners. The motors convert electrical energy to kinetic energy of rotation, and consist basically of a rigid current-carrying loop that rotates when placed in the field of a magnet.

Subtract the result of part (a) from the result of part (b):

$\Delta\Phi_B = 0.0499\ Wb - 0.0706\ Wb = -0.0207\ Wb$

Remarks: Notice that the rotation of the loop, not any change in the magnetic field, is responsible for the change in flux. This changing magnetic flux is necessary (essential) in the functioning (working) of electric motors and generators.

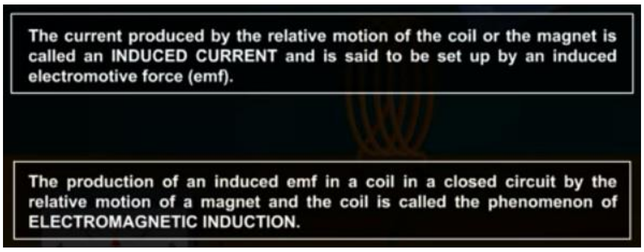
Exercise 3.1:(HW)

The loop, having rotated by 45°, rotates clockwise another 30°, so the normal to the plane points at an angle of 75° with respect to the direction of the magnetic field. **Find (a)** the magnetic flux through the loop when $\theta = 75°$ and **(b)** the change in magnetic flux during the rotation from 45° to 75°.

Answer: $(\boldsymbol{a})\ \mathbf{0.0183}\ \boldsymbol{Wb}$ $(\boldsymbol{b}) - \mathbf{0.0316}\ \boldsymbol{Wb}$

3.3 Faraday's Law of Induction

The usefulness (utility) of the concept of magnetic flux can be made obvious by another simple experiment that demonstrates the basic idea of electromagnetic induction. Consider a wire loop connected to an ammeter as in **Figure 3.8**. If a magnet is moved toward the loop, the ammeter reads a current in one direction, as in **Figure 3.8a**. When the magnet is held stationary (motionless), as in **Figure 3.8b**, the ammeter reads zero current. If the magnet is moved away from the loop, the ammeter reads a current in the opposite direction, as in **Figure 3.8c**. If the magnet is held stationary and the loop is moved either toward or away from the magnet, the ammeter also reads a current. From these observations, it can be concluded that a current is set up in the circuit as long as there is relative motion between the magnet and the loop. The same experimental results are found whether the loop moves or the magnet moves. We call such a current an induced current, because it is produced by an **Induced EMF** (Note the definitions below).



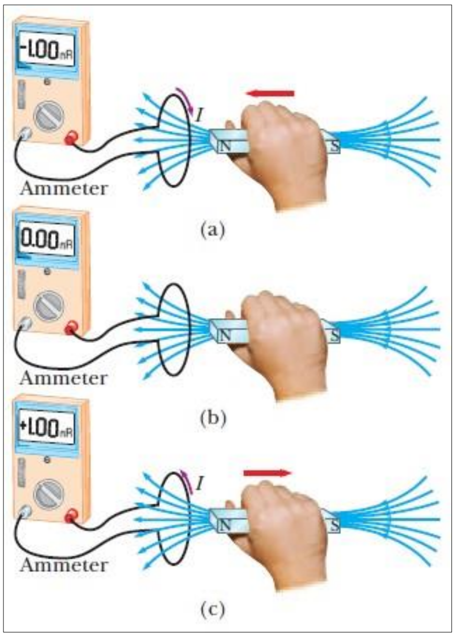


Figure 3.8: (a) When a magnet is moved toward a wire loop connected to an ammeter, the ammeter reads a current as shown, indicating that a current (I) is induced in the loop. **(b)** When the magnet is held stationary (motionless), no current is induced in the loop, even when the magnet is inside the loop. **(c)** When the magnet is moved away from the loop, the ammeter reads a current in the opposite direction, indicating an induced current going opposite the direction of the current in part **(a)**.

This experiment is similar to the Faraday experiment discussed in **Section 3.2**. In each case (a & c), an EMF is induced in a circuit when the magnetic flux through the circuit changes with time. It turns out that (it appears that) the instantaneous EMF induced in a circuit **equals** the negative of the rate of change of magnetic flux with respect to time through the circuit. This is Faraday's law of magnetic induction.

Faraday's law: If a circuit contains (N) tightly wound loops and the magnetic flux through each loop changes by the amount ($\Delta\Phi_B$) during the interval (Δt), the average EMF induced in the circuit during time (Δt) is.

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} \qquad (3.1)$$

Tip: Induced Current Requires a Change in Magnetic Flux

The existence of magnetic flux ($\Delta\Phi_B$) through an area is not sufficient to create an induced EMF. A change in the magnetic flux ($\Delta\Phi_B$) over some time interval (Δt) must occur for an EMF to be induced.

Because $\Phi_B = BA\cos\theta$, a change of any of the factors (B, A, or θ) with time produces an EMF. We study the effect of a change in each of these factors in the following sections. The minus sign in Equation 3.1 is included to indicate the polarity of the induced EMF. This polarity simply determines which of two different directions current will flow in a loop, a direction given by **Lenz's law**:

In physics, polarity is an attribute with two possible values. Polarity is a basic feature (ˈfēCHər) of the universe.

- An electric charge can have either positive or negative polarity.
- A voltage or potential difference between two points of an electric circuit has a polarity, describing which of the two points has the higher electric potential.
- A magnet has a polarity, in that it has two poles described as "north" and "south" pole.

Lenz's law: The current caused by the induced EMF travels in the direction that creates a magnetic field with flux opposing (əˈpōziNG) the change in the original flux through the circuit.

Lenz's law says that if the magnetic flux (original) through a loop is becoming more positive, then the induced EMF creates a current and associated magnetic field that produces negative magnetic flux. Some mistakenly think this "counter magnetic field" created by the induced current, called ($\vec{B}_{ind}$) ("ind" for induced) will always point in a direction opposite the applied magnetic field ($\vec{B}$), but this is only true half the time!

To understand Lenz's law, let us suppose a bar moving to the right on two parallel rails (rods) in the presence of a uniform magnetic field (External Field) that we shall refer to as the external magnetic field **(Figure 3.9a)**. As the bar moves to the right, the magnetic flux through the area enclosed by the circuit increases with time because the area increases.

Lenz's law states that the induced current must be directed so that the magnetic flux it produces opposes the change in the external magnetic flux. Because the external magnetic flux is increasing into the page, the induced current, if it is to oppose this change, must produce a flux ($\vec{B}_{ind}$) directed out of the page. Hence, the induced current must be directed counterclockwise when the bar moves to the right. (Use the RHR-2 to verify this direction).

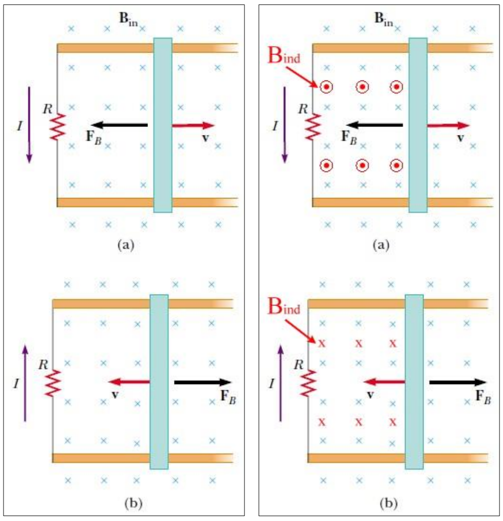


Figure 3.9: **(a)** As the conducting bar slides on the two fixed conducting rails, the magnetic flux through the area enclosed by the loop increases in time. By Lenz's law, the induced current must be counterclockwise so as to produce a counteracting magnetic flux directed out of the page. **(b)** When the bar moves to the left, the induced current must be clockwise. Why?

If the bar is moving to the left as shown in **Figure 3.9b**, the external magnetic flux through the area enclosed by the loop decreases with time. Because the flux is directed into the page, the direction of the induced current must be clockwise if it is to produce a flux ($\vec{B}_{ind}$) that also is directed into the page. In both case, the induced current tends to maintain the original flux through the area enclosed by the current loop.

Let us examine (study) this situation from the viewpoint of energy considerations. Suppose that the bar is given a slight push to the right. In the preceding analysis, we found that this motion sets up a counterclockwise current in the loop.

Let us see what happens if we assume that the current is clockwise in **Figure 3.9a**, such that the direction of the magnetic force exerted on the bar is to the right. This force would accelerate the bar and increase its velocity. This, in turn, would cause the area enclosed by the loop to increase more rapidly; this would result in an increase in the induced current, which would cause an increase in the force, which would produce an increase in the current, and so on. Actually, the system would acquire (obtain) energy with no additional input of energy. This is clearly inconsistent with all experience and with the law of conservation of energy. Thus, we are forced to conclude that the current must be counterclockwise.

The direction of the current in **Figure 3.9** can be determined by RHR-2: Point your right thumb in the direction that will cause the fingers on your right hand to curl in the direction of the induced field ($\vec{B}_{ind}$). In this case, that direction is counterclockwise: with the right thumb pointed in the direction of the current, your fingers curl down outside the loop and around and up through the inside of the loop.

Let us consider another situation, one in which a bar magnet moves toward a stationary metal loop, as shown in **Figure 3.10a**. As the magnet moves to the right toward the loop, the external magnetic flux through the loop increases with time. To counteract this increase in flux to the right, the induced current produces a flux to the left, as illustrated in **Figure 3.10b**; hence, the induced current is in the direction shown. Note that the magnetic field lines associated with the induced current oppose the motion of the magnet. Knowing that, like magnetic poles repel each other, we conclude that the left face of the current loop is in essence (ˈes(ə)ns) (essence ≡ intrinsic nature) a north pole and that the right face is a south pole.

If the magnet moves to the left, as shown in **Figure 3.10c**; its flux through the area enclosed by the loop, which is directed to the right, decreases in time. Now the induced current in the loop is in the direction shown in **Figure 3.10d**; because this current direction produces a magnetic flux in the same direction as the external flux. In this case, the left face of the loop is a south pole and the right face is a north pole.

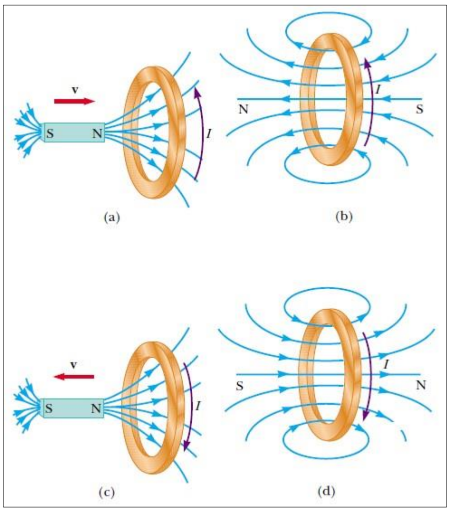


Figure 3.10: **(a)** When the magnet is moved toward the stationary conducting loop, a current is induced in the direction shown. **(b)** This induced current produces its own magnetic flux that is directed to the left and so counteracts the increasing external flux to the right. **(c)** When the magnet is moved away from the stationary conducting loop, a current is induced in the direction shown. **(d)** This induced current produces a magnetic flux that is directed to the right and so counteracts the decreasing external flux to the right.

Example 3.2:

A coil with 25 turns of wire is wrapped on a frame with a square cross section (1.8 cm) on a side. Each turn has the same area, equal to that of the frame, and the total resistance of the coil is (0.35 Ω). An applied uniform magnetic field is perpendicular to the plane of the coil, as in **Figure**. **(a)** If the field changes uniformly from (0 T to 0.5 T) in (0.8 s), **find** the induced EMF in the coil while the field is changing. **Find (b)** the magnitude and **(c)** the direction of the induced current in the coil while the field is changing.

Solution:

Chapter Four

Inductance

4.1 Self-Inductance

Consider a circuit consisting of a switch, a resistor, and a source of EMF, as in **Figure 4.1**. When the switch is closed, the current doesn't immediately change from zero to its maximum value (ε/R). The law of electromagnetic induction (Faraday's law) prevents this. What happens instead (inˈsted) is the following: as the current increases with time, the magnetic flux through the loop due to this current also increases. The increasing flux induces an EMF in the circuit that opposes (əˈpōz) the change in original magnetic flux. By Lenz's law, the induced EMF is in the direction indicated by the dashed battery in the **Figure**. The net potential difference across the resistor (R) is the EMF of the battery minus (-) the opposing induced EMF.

From Kirchhoff's law:

$$\varepsilon_{batt.} = \varepsilon_R \pm \varepsilon_{ind}$$

So:

$$\varepsilon_R = \varepsilon_{batt.} - \varepsilon_{ind}$$

As the magnitude of the current increases, the **rate** of increase lessens [(ˈles(ə)n) (reduces)] and hence the induced EMF decreases.

This opposing EMF results in a gradual increase in the current. For the same reason, when the switch is opened, the current doesn't immediately fall to zero. This effect is called self-induction because the changing flux through the circuit arises from the circuit itself. The EMF that is set up in the circuit is called a self-induced EMF.

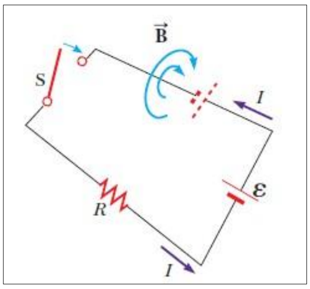


Figure 4.1: After the switch in the circuit is closed, the current produces its own magnetic flux through the loop. As the current increases towards its equilibrium value, the flux changes in time and induces an EMF in the loop. The battery drawn with dashed lines is a symbol for the self-induced EMF.

As a second example of self-inductance, consider **Figure 4.2**, which shows a coil wound (wo̅und) on a cylindrical iron core (A practical device would have several hundred turns). Assume that the current changes with time. When the current is in the direction shown, a magnetic field is set up inside the coil, directed from right to left **Figure 4.2a**. As a result, some lines of magnetic flux pass through the cross-sectional area of the coil. As the current changes with time, the flux through the coil changes and induces an EMF in the coil. Lenz's law shows that this induced EMF has a direction so as to oppose the change in the current. If the current is increasing, the induced EMF is as pictured in **Figure 4.2b**, and if the current is decreasing, the induced EMF is as shown in **Figure 4.2c**.

To evaluate self-inductance quantitatively, first note that, according to Faraday's law, the induced EMF is given by **Equation 3.1**:

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t},$$

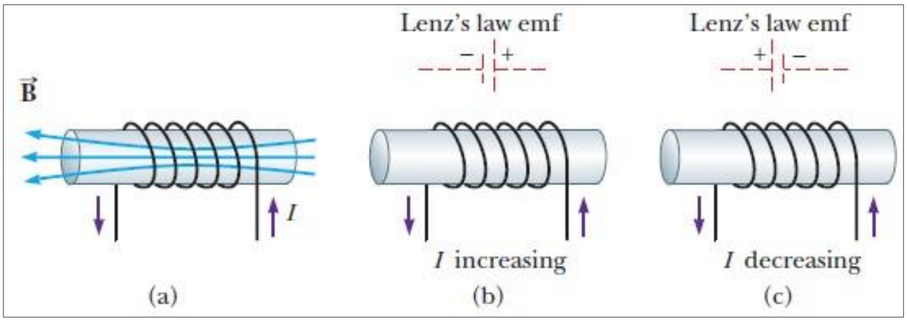


Figure 4.2: **(a)** A current in the coil produces a magnetic field directed to the left. **(b)** If the current increases, the coil acts as a source of EMF directed as shown by the dashed battery. **(c)** The induced EMF in the coil changes its polarity if the current decreases.

The magnetic flux is proportional to the magnetic field, which is proportional to the current in the coil. Therefore, the self-induced EMF must be proportional to the rate of change of the current with time, or:

$$\varepsilon = -L\frac{\Delta I}{\Delta t} \qquad (4\text{-}1)$$

Where (L) is a proportionality constant called the inductance of the device. The negative sign indicates that a changing current induces an EMF in opposition to the change. This means that if the current is increasing $(\Delta I\ positive)$, the induced EMF is negative, indicating opposition to the increase in current. Likewise (similarly), if the current is decreasing $(\Delta I\ negative)$, the sign of the induced EMF is positive, indicating that the EMF is acting to oppose the decrease.

The inductance of a coil depends on the cross-sectional area of the coil and other quantities, all of which can be grouped under the general heading of geometric factors. The SI unit of inductance is the **henry** (H), which, from **Equation 4.1**, is equal to 1 volt-second per ampere:

$$1\ \text{H} = 1\ \text{V.s/A}$$

In the process of calculating self-inductance, it is often convenient to equate **Equations 3.1 and 4.1** to find an expression for L:

$$N\frac{\Delta\Phi_B}{\Delta t} = L\frac{\Delta I}{\Delta t}$$

$$L = N\frac{\Delta\Phi_B}{\Delta I} = \frac{N\Phi_B}{I}, \qquad (4.2) \qquad \text{(Inductance)}$$

In some circuits, a spark occurs between the poles of a switch when the switch is opened (turn off). Why isn't there a spark when the switch for this circuit is closed (turn on)?

Explanation: according to Lenz's law, the direction of induced EMF is such that the induced magnetic field opposes change in the original magnetic flux. When the switch is opened, the sudden drop in the magnetic field in the circuit induces an EMF in a direction that opposes change in the original current. This induced EMF can cause a spark as the current bridges the air gap between the poles of the switch. The spark doesn't occur when the switch is closed, because the original current is zero and the induced EMF opposes any change in that current.

In general, determining (diˈtərminiNG) the inductance of a given current element can be challenging. Finding an expression for the inductance of a common solenoid, however, is straightforward (strātˈfôrwərd) (simple). Let the solenoid have ($N\ turns$) and length (l). Assume that (l) is large compared with the radius and that the core of the solenoid is air. We take the interior magnetic field to be uniform and given by **Equation 1.24**:

$$B = \mu_\circ nI = \mu_\circ \frac{N}{l} I$$

where ($n = N/l$) is the number of turns per unit length. The magnetic flux through each turn is therefore:

$$\Phi_B = BA = \mu_\circ \frac{N}{l} IA$$

where (A) is the cross-sectional area of the solenoid. From this expression and **Equation 4.2**, we find that:

$$L = \frac{N\Phi_B}{I} = \frac{\mu_\circ N^2 A}{l} \qquad (4.3a)$$

This shows that (L) depends on the geometric factors (l) and (A) and on $(\mu_\circ)$ and is proportional to the square of the number of turns. Because $(N = nl)$, we can also express the result in the form:

$$L = \mu_\circ \frac{(nl)^2}{l} A = \mu_\circ n^2 A l = \mu_\circ n^2 V \qquad (4.3b)$$

Where $(V = Al)$ is the volume of the solenoid.

Example 4.1: Inductance, Self-Induced EMF, and Solenoids

Goal: Calculate the inductance and self-induced EMF of a solenoid.

(a) Calculate the inductance of a solenoid containing $(300\ turns)$ if the length of the solenoid is $(25\ cm)$ and its cross-sectional area is $(4 \times 10^{-4}\ m^2)$. **(b)** Calculate the self-induced EMF in the solenoid described in (a) if the current in the solenoid decreases at the rate of $(50\ A/s)$.

Solution:

(a) Calculate the inductance of the solenoid.

$$L = \frac{\mu_\circ N^2 A}{l} = (4\pi \times 10^{-7}\ T.m/A)\frac{(300)^2(4\times10^{-4}\ m^2)}{(25\times10^{-2}\ m)} = 1.81 \times 10^{-4}\ T.m^2/A = 0.181\ mH$$

(b) Calculate the self-induced EMF in the solenoid.

$$\varepsilon = -L\frac{\Delta I}{\Delta t} = -(1.81 \times 10^{-4}\ H)(-50\ A/s) = 9.05\ mV.$$

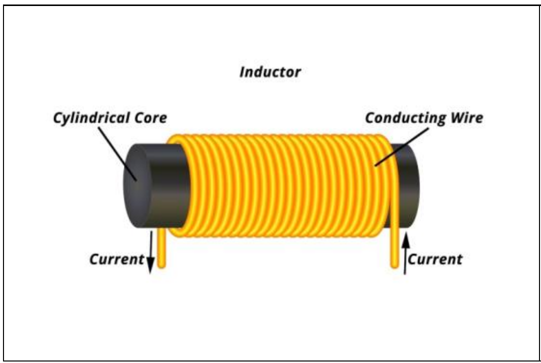
Remark: Notice that $\left(\frac{\Delta I}{\Delta t}\right)$ is negative because the current is decreasing with time.

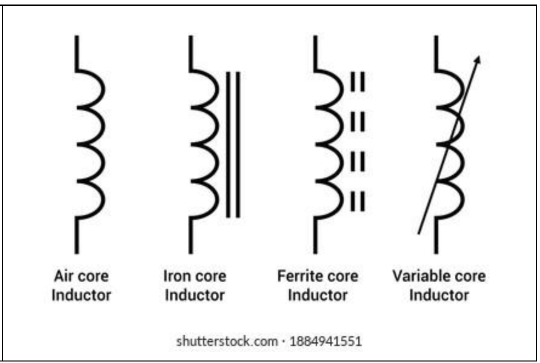
Exercise 4.1:(HW)

A solenoid is to have an inductance of $(0.285\ mH)$, a cross-sectional area of $(6 \times 10^{-4}\ m^2)$, and a length of $(36\ cm)$. **(a)** How many turns per unit length should it have? **(b)** If the self-induced EMF is $(-12.5\ mV)$ at a given time, at what rate is the current changing at that instant?

Answer: ($\boldsymbol{a}$) 1025 turns/m ($\boldsymbol{b}$) 43.9 A/s

Inductor: also called a coil, choke, or reactor, is a passive two-terminal electrical component that stores energy in a magnetic field when electric current flows through it. An inductor typically consists of an insulated wire wound into a coil as in the Figure below.





4.2 Energy Stored In Magnetic Field

The EMF induced by an inductor prevents a battery from establishing an instantaneous current in a circuit. The battery has to do work to produce a current. We can think of this needed work as energy stored in the inductor in its magnetic field. We find that the energy stored by an inductor is:

$$PE_L = \frac{1}{2}LI^2, \qquad (4.4) \qquad \text{(Energy stored in an inductor)}$$

Note that the result is similar in form to the expression for the energy stored in a charged capacitor ~~(Equation 16.18)~~:

$$PE_C = \frac{1}{2}C(\Delta V)^2, \qquad (4.5) \qquad \text{(Energy stored in a capacitor)}$$

Example 4.2: Magnetic Energy

Goal: Relate the storage of magnetic energy to currents in an RL circuit.

A $(12\ V)$ battery is connected in series to a $(25\ \Omega)$ resistor and a $(5\ H)$ inductor. (a) Find the maximum current in the circuit. (b) Find the energy stored in the inductor at this time. (c) How much energy is stored in the inductor when the current is changing at a rate of $(1.5\ A/s)$?

Strategy: In part (a), Ohm's law and Kirchhoff's voltage rule yield (give) the maximum current, because the voltage across the inductor is zero when the current is maximal. Substituting the current into **Equation 4.4**

Chapter Five

Alternating Current Circuits and Electromagnetic Waves

5.2 Kirchhoff's Rules and Complex DC Circuits

We can analyze simple circuits using Ohm's law and the rules for series and parallel combinations of resistors. However, there are many ways in which resistors can be connected so that the circuits formed can't be reduced to a single equivalent resistor. The procedure for analyzing more complex circuits can be facilitated by the use of **two simple rules** called Kirchhof's rules (Serway):

1) Kirchhoff's Current Law (KCL):
   - KCL (1^{st} Law) **states** that the current flowing into a node (or a junction) must be equal to the current flowing out of it. Kirchhoff's first law is based on the conservation of charge because sum of current entering to the junction (node) is equal to sum of current leaving the junction.
   - This idea by Kirchhoff is commonly known as the Conservation of Charge.
2) Kirchhoff's Voltage Law (KVL):
   - KVL (2^{nd} Law) **states** that the algebraic sum of potential drops in a closed circuit is zero. So, it is based on the conservation of energy.
   - This idea by Kirchhoff is known as the Conservation of Energy.

According to KVL, The voltage around a loop equals the sum of every voltage drop in the same loop for any closed network and equals zero. Put differently (In other words), the algebraic sum of every voltage in the loop has to be equal to zero and this property of Kirchhoff's law is called conservation of energy.

> 1. The sum of the currents entering any junction must equal the sum of the currents leaving that junction. (This rule is often referred to as the **junction rule**.)
> 2. The sum of the potential differences across all the elements around any closed circuit loop must be zero. (This rule is usually called the **loop rule**.)

The junction rule (KCL) is a statement (term) of conservation of charge. Whatever current enters a given point in a circuit must leave that point because charge can't build up (accumulate) or disappear at a point. If we apply this rule to the junction (node) in Figure below, we get:

$$I_1 = I_2 + I_3$$

Figure 18.12b represents a mechanical analog of the circuit shown in Figure 18.12a. In this analog, water flows through a branched pipe with no leaks. The flow rate into the pipe equals the total flow rate out of the two branches.

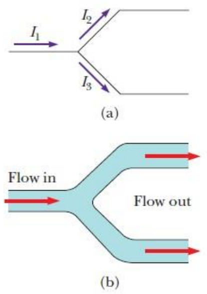
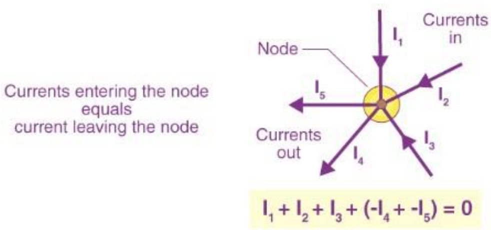
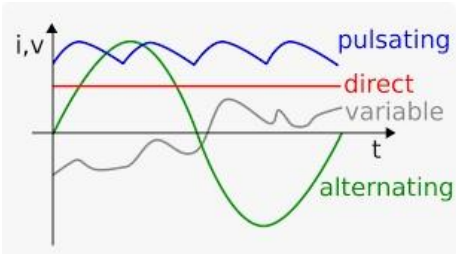


Figure 18.12 (a) A schematic diagram illustrating Kirchhoff's junction rule. Conservation of charge requires that whatever current enters a junction must leave that junction. In this case, therefore, $I_1 = I_2 + I_3$. (b) A mechanical analog of the junction rule: the net flow in must equal the net flow out.





The loop rule (KVL) is equivalent to the principle of conservation of energy. Any charge that moves around any closed loop in a circuit (starting and ending at the same point) must gain as much energy as it loses.

It gains energy as (when) it is pumped through a source of EMF. Its energy may decrease in the form of a potential drop -IR across a resistor or as a result of flowing backward through a source of EMF, from the positive to the negative terminal inside the battery. In the latter case, electrical energy is converted to chemical energy as the battery is charged. When applying Kirchhoff's rules, you must make two decisions at the beginning of the problem:

1) Assign (determine) symbols and directions to the currents in all branches of the circuit. Don't worry about guessing (estimate) the direction of a current incorrectly; the resulting answer will be negative, but its magnitude will be correct. (This is because the equations are linear in the currents - all currents are to the first power).
   - A linear equation is an algebraic equation in which each variable term is raised to the exponent or power of **1**. A linear equation in one or two variables always represents a straight line when graphed.

- **Example:** x+2y = 4 is a linear equation and the graph of this linear equation is a straight line.

2) When applying the loop rule (KVL), you must choose a direction for traversing (passing) the loop, and be consistent in going either clockwise or counterclockwise. As you traverse the loop, record voltage drops and rises according to the following rules (summarized in Figure 18.13, where it is assumed that **movement** is from point **a** toward point **b**):

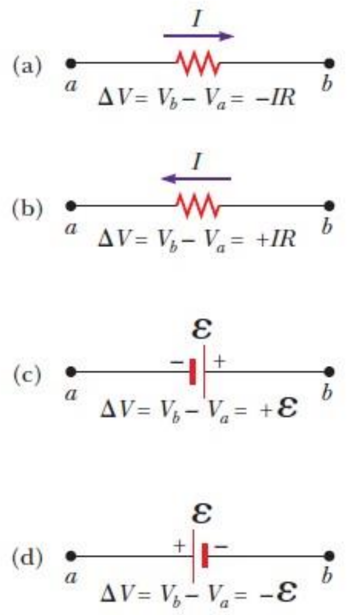


Figure 18.13 Rules for determining the potential differences across a resistor and a battery, assuming the battery has no internal resistance.

a) If a resistor is traversed in the direction of the current, the change in electric potential across the resistor is -IR (Fig. 18.13a).
b) If a resistor is traversed in the direction opposite the current, the change in electric potential across the resistor is +IR (Fig. 18.13b).
c) If a source of emf is traversed in the direction of the emf (from - to + on the terminals), the change in electric potential is +ε (Fig. 18.13c).
d) If a source of emf is traversed in the direction opposite the emf (from + to - on the terminals), the change in electric potential is -ε (Fig. 18.13d).

There are limits to the number of times the junction rule (KCL) and the loop rule (KVL) can be used. You can use the junction rule as often as needed as long as, each time you write an equation, you include in it a current that has not been used in a previous junction-rule equation. (If this procedure isn't followed, the new equation will just be a combination of two other equations that you already have). In general, the number of times the junction rule can be used is one fewer than the number of junction points in the circuit.

The loop rule (KVL) can also be used as often as needed, so long as a new circuit element (resistor or battery) or a new current appears in each new equation. To solve a particular circuit problem, you need as numerous independent equations as you have unknowns.

Problem-Solving Strategy Applying Kirchhoff's Rules to a Circuit

After taking all the above comments into consideration, we will now proceed:

1) Solve the equations simultaneously for the unknown quantities, using substitution or any other method familiar to the student.
2) Check your answers by substituting them into the original equations.

GUSTAV KIRCHHOFF, German Physicist (1824–1887)

Together with German chemist Robert Bunsen, Kirchhoff, a professor at Heidelberg, invented the spectroscopy that we study in Chapter 28. He also formulated another rule that states, "A cool substance will absorb light of the same wavelengths that it emits when hot."

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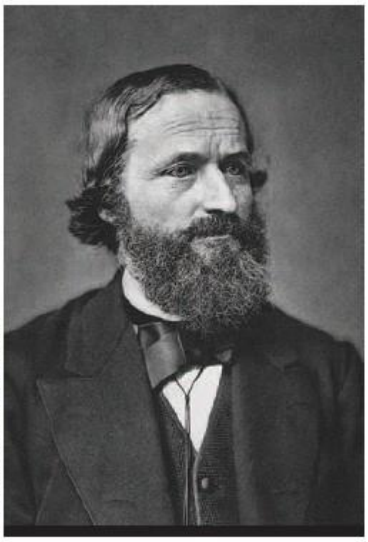
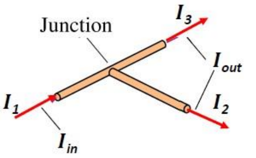


Kirchhoff's Junction Law

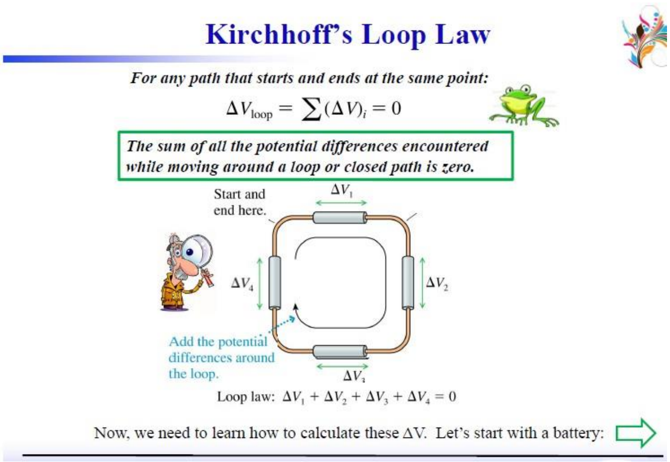
For a junction, the law of conservation of current requires that:

$$\sum I_{in} = \sum I_{out}$$

At any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction.



Junction law: $I_1 = I_2 + I_3$



The sum of all potential differences encountered while moving around a loop or closed path is zero.