

Chapter One :Review of probability theory

Definition(1):Random experiment

A random experiment is an experiment that can be repeated under the same conditions and whose result cannot be predicted with certainty.

Example(1):We consider the following three classical experiments:

E_1 : a coin is tossed three times and the number of "tails" obtained is recorded.

E_2 : a die is thrown until a "6" appears, and the number of throws made is counted;

E_3 : a number is taken at random in the interval (0,1).

Definition(2):Sample Space

The sample space S of a random experiment is the set of all possible outcomes of this experiment.

Example(2) The sample spaces that correspond to the random experiments in the example above are the following:

$$S_1 = \{0,1,2,3\};$$

$$S_2 = \{1,2,\dots\};$$

$$S_3 = (0,1).$$

Definition(3):Event

An event is a subset of the sample space S . In particular, each possible outcome of a random experiment is called an elementary event.

The number of elementary events in a sample space may be finite (S_1), countably infinite (S_2), or uncountable infinite (S_3).

Notations

Union: $A \cup B$ (corresponds to the case when we seek the probability that one event or another one occurred, or that both events occurred).

Intersection: $A \cap B$ or AB (when we seek the probability that an event and another one occurred). If two events are incompatible (or mutually exclusive) then we write that $A \cap B = \emptyset$ (the empty set).

Complement: A^c (the set of elementary events that do not belong to A).

Inclusion: $A \subset B$ (when all the elementary events that belong to A also belong to B).

Definition(4):Probability measure

A probability measure is a function P of the subsets of a sample space S , associated with a random experiment E , that possesses the following properties: