Chapter four: Poisson Process

4.1 Counting Process

A stochastic process {X(t), $t \geq 0$} is a counting process if X(t) represents the total number of events that have occur in the interval (0,t]. For example:

- Number of people who were born by time t.
- Number of persons entering a store before time t.

then X(t) should satisfy:

1. $X(t) \geq 0$ *and* $X(0) = 0$
2. $X(t)$ is integer valued
3. If s<t ,then $X(s) \leq X(t)$
4. For s<t , $X(t) - X(s)$ equals the number of events that have occurred on the interval (s,t].

Remarks:

- A counting process X(t) is said to be independent increments if the numbers of events which occure in disjoint intervals are independent.
- A counting process X(t) is said to be stationary increments process if the number of events in the interval (s+h,t+h) that is (X(t+h)-X(s+h)) has the same distribution as the number of events in the interval (s,t] that is ((X(t)-X(s)) for all s<t ,h>0.