

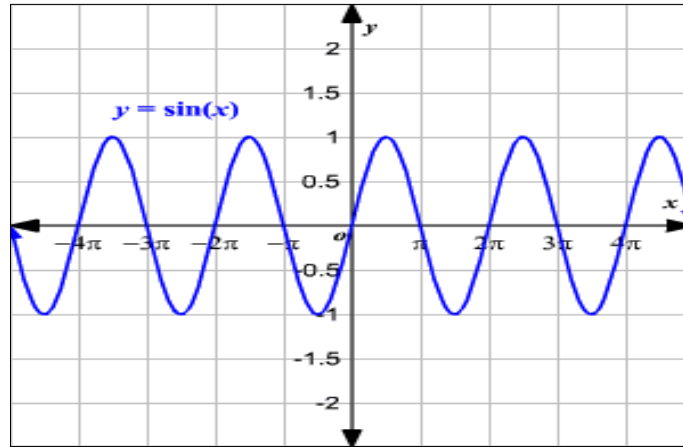
Trigonometric Functions الدوال المثلثية

$\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$, $\csc x$

Sine Function: $y = f(x) = \sin x$

دالة الجيب

x	$y(x) = \sin x$
$-\pi$	$\sin(-\pi) = 0$
$-\frac{\pi}{2}$	$\sin(-\frac{\pi}{2}) = -1$
0	$\sin(0) = 0$
$\frac{\pi}{2}$	$\sin(\frac{\pi}{2}) = 1$
π	$\sin(\pi) = 0$



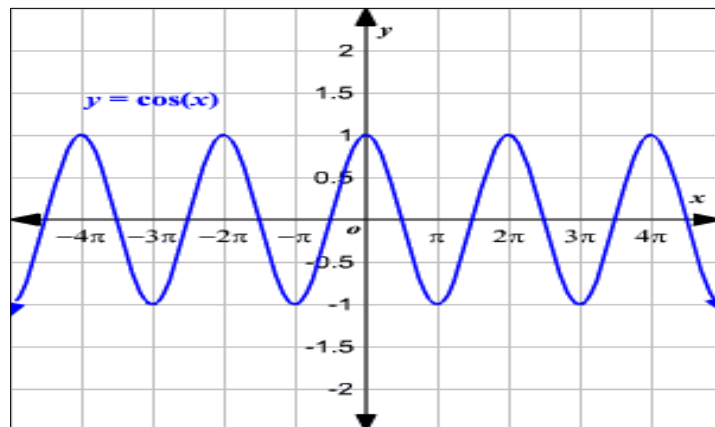
Properties:

- Domain: all real numbers
- Range: $[-1, 1]$
- Period = 2π . That is $\sin(x + 2\pi) = \sin(x)$
- $y = \sin x$ is an odd function. That is $\sin(-x) = -\sin(x)$

Cosine Function: $y = f(x) = \cos x$

دالة جيب تمام

x	$y(x) = \cos x$
$-\pi$	$\cos(-\pi) = -1$
$-\frac{\pi}{2}$	$\cos(-\frac{\pi}{2}) = 0$
0	$\cos(0) = 1$
$\frac{\pi}{2}$	$\cos(\frac{\pi}{2}) = 0$
π	$\cos(\pi) = -1$

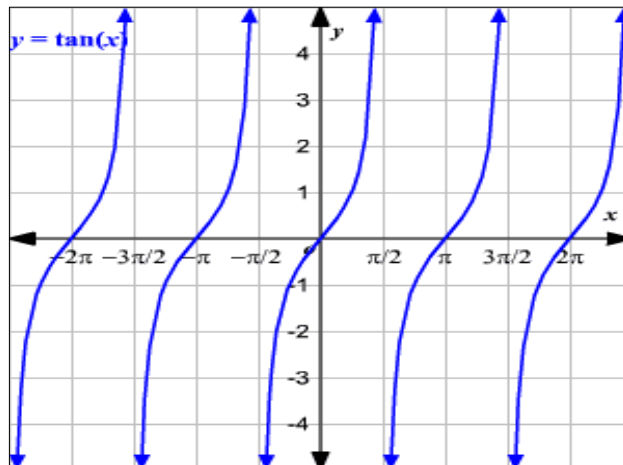


Properties:

- Domain: all real numbers
- Range: $[-1, 1]$
- Period = 2π . That is $\cos(x + 2\pi) = \cos(x)$
- $y = \cos x$ is an even function. That is $\cos(-x) = \cos(x)$

Tangent Function: $y = f(x) = \tan x = \frac{\sin x}{\cos x}$, $\cos x \neq 0$ دالة الظل:

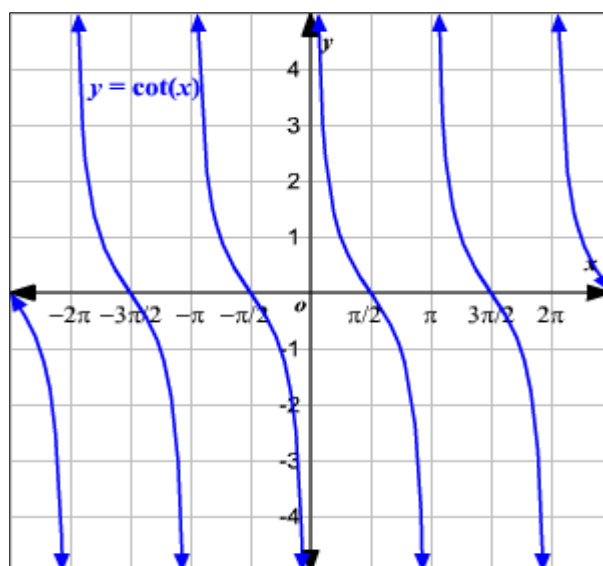
x	$y(x) = \tan x$
$-\pi$	$\tan(-\pi) = 0$
$-\frac{\pi}{4}$	$\tan\left(-\frac{\pi}{4}\right) = -1$
0	$\tan(0) = 0$
$\frac{\pi}{4}$	$\tan\left(\frac{\pi}{4}\right) = 1$
π	$\tan(\pi) = 0$
2π	$\tan(2\pi) = 0$



Properties:

- Domain: all real numbers except $\frac{\pi}{2} + n\pi$, n is an integer.
- Range: all real numbers
- Period = π . That is $\tan(x + \pi) = \tan(x)$
- $y = \tan x$ is an odd function. That is $\tan(-x) = -\tan(x)$

Cotangent Function: $y = f(x) = \cot x = \frac{\cos x}{\sin x}$, $\sin x \neq 0$ دالة ظل التمام:

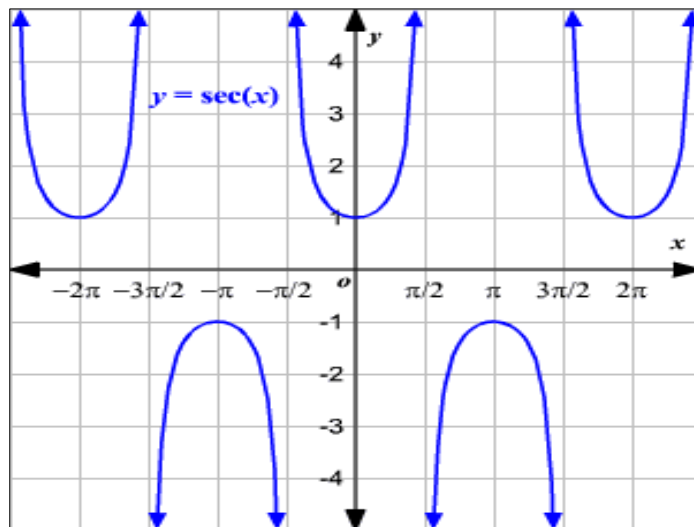


Properties:

- Domain: all real numbers except $n\pi$, n is an integer.
- Range: all real numbers
- Period = π . That is $\cot(x + \pi) = \cot(x)$
- $y = \cot x$ is an odd function. That is $\cot(-x) = -\cot(x)$

Secant Function: $y = f(x) = \sec x = \frac{1}{\cos x}$, $\cos x \neq 0$

دالة القاطع:

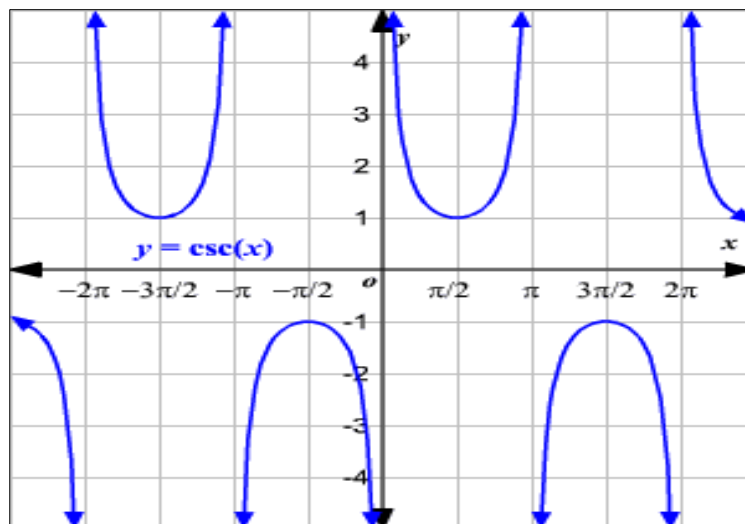


Properties:

- Domain: all real numbers except $\frac{\pi}{2} + n\pi$, n is an integer.
- Range: $(-\infty, -1] \cup [1, \infty)$
- Period = 2π . That is $\sec(x + 2\pi) = \sec(x)$
- $y = \sec x$ is an even function. That is $\sec(-x) = \sec(x)$

Cosecant Function: $y = f(x) = \csc x = \frac{1}{\sin x}$, $\sin x \neq 0$

دالة قاطع التمام:



Properties:

- Domain: all real numbers except $n\pi$, n is an integer.
- Range: $(-\infty, -1] \cup [1, \infty)$
- Period = 2π . That is $\csc(x + 2\pi) = \csc(x)$
- $y = \csc x$ is an odd function. That is $\csc(-x) = -\csc(x)$

غايات بعض الدوال:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

بعض القوانين (المتطابقات):

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

$$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$$

$$\sec^2 \alpha = 1 + \tan^2 \alpha$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

مشتقات الدوال المثلثية: إذا كانت u دالة قابلة للاشتقاق بالنسبة إلى x فإن:

$$\frac{d}{dx} (\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx} (\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx} (\tan u) = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx} (\cot u) = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx} (\sec u) = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx} (\csc u) = -\csc u \cot u \frac{du}{dx}$$

مثال: لتكن $y = \sin(2x^3 - 3)$ ، فإن

$$y' = \frac{dy}{dx} = \cos(2x^3 - 3) \cdot (6x^2) = 6x^2 \cos(2x^3 - 3)$$

مثال: جد مشتقة الدالة $y = \cos(2\theta^3 - 3\theta^{-2})$

$$\begin{aligned} y' = \frac{dy}{d\theta} &= -\sin(2\theta^3 - 3\theta^{-2}) \cdot (6\theta^2 + 6\theta^{-3}) \\ &= -(6\theta^2 + 6\theta^{-3}) \cdot \sin(2\theta^3 - 3\theta^{-2}) \end{aligned}$$

مثال: جد مشتقة الدالة $y = \tan(x^{-2})$

الحل:

$$y' = \frac{dy}{dx} = \sec^2(x^{-2}) \cdot (-2x^{-3}) = -2x^{-3} \cdot \sec^2(x^{-2})$$

مثال: جد مشتقة الدالة $y = \cot(3x)$

الحل:

$$y' = \frac{dy}{dx} = -\csc^2(3x) \cdot (3) = -3 \cdot \csc^2(3x)$$

مثال: جد مشتقة الدالة $y = \sec(\theta^2)$

الحل:

$$y' = \frac{dy}{d\theta} = \sec(\theta^2) \cdot \tan(\theta^2) \cdot (2\theta) = 2\theta \cdot \sec(\theta^2) \cdot \tan(\theta^2)$$

مثال: جد مشتقة الدالة $y = \csc(x^3)$

الحل:

$$y' = \frac{dy}{dx} = -\csc(x^3) \cdot \cot(x^3) \cdot (3x^2) = -3x^2 \csc(x^3) \cdot \cot(x^3)$$

مثال: جد مشتقة الدالة $y = \csc(2x^5 - 3)$

الحل:

$$\begin{aligned} y' = \frac{dy}{dx} &= -\csc(2x^5 - 3) \cdot \cot(2x^5 - 3) \cdot (10x^4) \\ &= -10x^4 \cdot \csc(2x^5 - 3) \cdot \cot(2x^5 - 3) \end{aligned}$$

مثال: جد مشتقة الدالة $y = x \cdot \tan\left(\frac{1}{x}\right)$
الحل:

$$\begin{aligned} y' = \frac{dy}{dx} &= x \cdot \left(\sec^2\left(\frac{1}{x}\right) \cdot \left(\frac{-1}{x^2}\right) \right) + \tan\left(\frac{1}{x}\right) \cdot 1 \\ &= \frac{-1}{x} \cdot \sec^2\left(\frac{1}{x}\right) + \tan\left(\frac{1}{x}\right) \end{aligned}$$

الواجب: أحسب المشتقة للدوال الآتية:

Find the derivative of the following functions.

$$y = \sin^5(3x^2) \quad , \quad y = \sec((2x + 1)^{5/2}) \quad , \quad y = \frac{1}{x^2 \sin^3(x)}$$

$$y = (x^4 - \cot(x))^3 \quad , \quad y = \sqrt{1 + \cos^2(x)} \quad , \quad y = (\sin(x) - \cos(x))^2$$

$$y = \sin^3(2x) \quad , \quad y = x^2 \sin x + 2x \cos x - 2 \sin x \quad , \quad y = \sqrt{x} \tan x$$

$$y = \cos(1 - 2x) \quad , \quad y = \cos(2x + 1) \quad , \quad y = \tan x + 2 \sin(x^2) - 7x$$

$$y = \tan(\cos x) - 3 \cos^2(5x^2 - 7) \quad , \quad y = (\tan x + 2x)^2$$

$$y = \sin(2x^3) \quad , \quad y = \cos(\sqrt{x}) \quad , \quad y = 4 \tan(-2x^2 - 3) \quad , \quad y = \cos\left(\frac{1}{x}\right)$$

Find the value of $f'\left(\frac{\pi}{6}\right)$ for $f(x) = \cos(2x)$

Find the value of $f'\left(\frac{\pi}{2}\right)$ for $f(x) = 5 \cos x + 3x - 1$

Find the value of $f'\left(\frac{\pi}{4}\right)$ for $f(x) = \frac{1}{2} \sin(2x) - 4$

تكمال الدوال المثلثية

1	$\int \sin x \, dx = -\cos x + C$	6	$\int \csc x \, dx = \ln \csc x - \cot x + C$
2	$\int \cos x \, dx = \sin x + C$	7	$\int \sec^2 x \, dx = \tan x + C$
3	$\int \tan x \, dx = \ln \sec x + C$	8	$\int \csc^2 x \, dx = -\cot x + C$
4	$\int \cot x \, dx = -\ln \csc x + C$	9	$\int \sec x \cdot \tan x \, dx = \sec x + C$
5	$\int \sec x \, dx = \ln \sec x + \tan x + C$	10	$\int \csc x \cdot \cot x \, dx = -\csc x + C$

Example: Evaluate the integral $\int \cos(3x) dx$

الحل: بالضرب والقسمة على 3 ، فأن

$$\frac{3}{3} \int \cos(3x) dx = \frac{1}{3} \int 3 \cdot \cos(3x) dx = \frac{1}{3} \sin(3x) + C$$

مثال: جد ناتج التكامل $\int \tan^2(2x) dx$

الحل: باستعمال المتطابقة $\tan^2(2x) = \sec^2(2x) - 1$

$$\begin{aligned} \int \tan^2(2x) dx &= \int (\sec^2(2x) - 1) dx = \int \sec^2(2x) dx - \int dx \\ \frac{2}{2} \int \sec^2(2x) dx - \int dx &= \frac{1}{2} \int 2 \cdot \sec^2(2x) dx - \int dx \\ &= \frac{1}{2} \tan(2x) - x + C \end{aligned}$$

مثال: جد ناتج التكامل $\int x \cdot \tan(2x^2 + 1) dx$

الحل: $\int x \cdot \tan(2x^2 + 1) dx = \frac{1}{4} \int 4x \cdot \tan(2x^2 + 1) dx$

$$= \frac{1}{4} \ln|\sec(2x^2 + 1)| + C$$

مثال: جد ناتج التكامل $\int \cot\left(7 - \frac{x}{2}\right) dx$

الحل: $\int \cot\left(7 - \frac{x}{2}\right) dx = \frac{1}{(-1/2)} \int (-1/2) \cot\left(7 - \frac{x}{2}\right) dx$

$$= \frac{1}{(-1/2)} \left(-\ln \left| \csc\left(7 - \frac{x}{2}\right) \right| \right) + C$$

$$= 2 \ln \left| \csc\left(7 - \frac{x}{2}\right) \right| + C$$

مثال: جد ناتج التكامل $\int \frac{\cos(2x)}{\sin^3(2x)} dx$

الحل:

$$\begin{aligned} \int \frac{\cos(2x)}{\sin^3(2x)} dx &= \int (\sin^{-3}(2x)) \cdot (\cos(2x)) \cdot dx \\ &= \frac{1}{2} \int (\sin^{-3}(2x)) \cdot (2 \cdot \cos(2x)) \cdot dx = \frac{1}{2} \cdot \frac{\sin^{-3+1}(2x)}{-3+1} + C \\ &= \frac{1}{2} \cdot \frac{\sin^{-2}(2x)}{-2} + C = \frac{-1}{4} \frac{1}{\sin^2(2x)} + C = -\frac{1}{4} \csc^2(2x) + C \end{aligned}$$

الواجب: أحسب التكاملات الآتية:

$$\begin{aligned} & \int (\sin(3x + 2) + \cos(2 - 3x)) dx, \quad \int \sec^2(4x) dx \\ & \int x^2 \csc^2\left(3 - \frac{x^3}{3}\right) dx, \quad \int x^2 \csc(2x^3) \cdot \cot(2x^3) dx \\ & \int \frac{3 \sin(2 - \sqrt{x})}{\sqrt{x}} dx, \quad \int (\sec x + \tan x)^2 dx, \quad \int \frac{\cos(3 + 5 \ln(9x))}{7x} dx \\ & \int \cos(6x) \cdot \cos(9 + 4 \sin(6x)) dx, \quad \int \frac{\tan\left(5 - \frac{4}{\sqrt{x}}\right)}{\sqrt{x^3}} dx \\ & \int \sin(4x) dx, \quad \int \frac{3 \sin(2 - \sqrt{x})}{\sqrt{x}} dx, \quad \int \frac{\tan\left(5 - \frac{4}{\sqrt{x}}\right)}{\sqrt{x^3}} dx \end{aligned}$$

الدوال المثلثية العكسية

$\sin^{-1}(x)$	$\cos^{-1}(x)$	$\tan^{-1}(x)$	$\cot^{-1}(x)$	$\sec^{-1}(x)$	$\csc^{-1}(x)$
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Or

$\arcsin x$	$\arccos x$	$\arctan x$	$\operatorname{arccot} x$	$\operatorname{arcsec} x$	$\operatorname{arccsc} x$
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$$\sin^{-1}(x) \neq \frac{1}{\sin(x)}$$

ملاحظة:

مشتقات الدوال المثلثية العكسية: إذا كانت u دالة قابلة للاشتقاق بالنسبة إلى x فإن:

$$\frac{d}{dx}(\sin^{-1}(u)) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx}(\cos^{-1}(u)) = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx}(\tan^{-1}(u)) = \frac{1}{1+u^2} \frac{du}{dx}$$

$$\frac{d}{dx}(\cot^{-1}(u)) = \frac{-1}{1+u^2} \frac{du}{dx}$$

$$\frac{d}{dx}(\sec^{-1}(u)) = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

$$\frac{d}{dx}(\csc^{-1}(u)) = \frac{-1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$