Chapter one

"Random Variables"

1– Th concept of Random variables مفهوم المتغيرات العشوائية

Definitions:

Given a random experiment with sample space $\boldsymbol{S}$, A random variable (r.v.) X is a function which assigns to each element $\boldsymbol{w} \in \boldsymbol{S}$ a real number $\boldsymbol{X}(\boldsymbol{w})$ in the set $\boldsymbol{E}$.

Ex) a _ Suppose that an experiment consists of tossing two fair coins :

SoL) $S = \{HH, HT, TH, TT\}$ _____ random experiment

Random variables

X : تمثل عدد مرات ظهور (H)

X= 0 , 1 , 2

Y : تمثل عدد مرات ظهور (T)

Y= 0 , 1 , 2

$\boldsymbol{P(X=x)}$

$\boldsymbol{P(X=0)=\frac{1}{4}}$

$\boldsymbol{P(X=1)=\frac{2}{4}}$

$\boldsymbol{P(X=2)=\frac{1}{4}}$

$\longrightarrow \boldsymbol{\sum P(X=x_i)=1}$

$\underline{X}$ Values

$\boldsymbol{HH \to 2}$

$\boldsymbol{HT \to 1}$

$\boldsymbol{TH \to 1}$

$\boldsymbol{TT \to 0}$

b _ Suppose that an experiment consists of tossing three fair coins :-

Sol/ $S = \{\underbrace{HHH}_{3}, \underbrace{HHT, HTH, THH}_{2}, \underbrace{TTH, THT, HTT}_{1}, \underbrace{TTT}_{0}\}$

$X = 0, 1, 2, 3$ X تمثل عدد مرات ظهور الصورة (H)

$P(X=0) = 1/8$

$P(X=1) = 3/8$

$P(X=2) = 3/8$

$P(X=3) = 1/8$

$\sum p(x) = 1$

$3 \leftarrow HHH$

$2 \leftarrow \{HHT, HTH, THH\}$

$1 \leftarrow \{TTH, THT, HTT\}$

$0 \leftarrow TTT$

c _ Suppose that an experiment consists of tossing a pair of unbiased dice , X be a random variable indicating the sum of the two faces :-

Sol/

X =	2	3	4	5	6	7	8	9	10	11	12
	(1,1)	(1,2)	(1,3)	(1,4)	(15)	(16)	(26)	(36)	(46)	(56)	(66)
		(21)	(22)	(23)	(24)	(25)	(35)	(45)	(55)	(65)	
			(31)	(32)	(33)	(34)	(44)	(54)	(64)		
				(41)	(42)	(43)	(53)	(63)			
					(51)	(52)	(62)				
						(6,1)					

$$P(X=x)=\frac{1}{36},\frac{2}{36},\frac{3}{36},\frac{4}{36},\frac{5}{36},\frac{6}{36},\frac{5}{36},\frac{4}{36},\frac{3}{36},\frac{2}{36},\frac{1}{36}$$

X	2	3	4	5	6	7	8	9	10	11	12
P (X=x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

And $\sum \boldsymbol{P}(\boldsymbol{X}=\boldsymbol{x_i})=\mathbf{1}$

ويمكن تمثيل (r.v.) وفق دوال وعلى النحو الاتي :-

$$P(X)\begin{cases} 0 & x=0 \\ \frac{x-1}{36} & x=2,3,4,5,6,7 \\ \frac{13-x}{36} & x=8,9,10,11,12 \\ 1 & \Sigma P(X)=1 \end{cases}$$

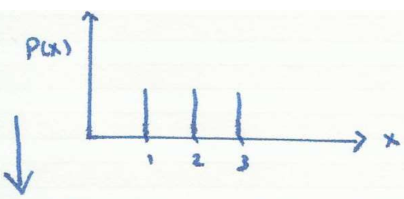
ويمكن تمثيل دالة خطية عند رمي زار مرة واحدة

$S = \{1, 2, 3, 4, 5, 6\}$

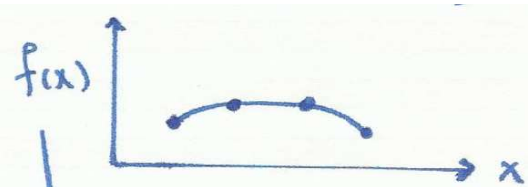
$$P(x) = \begin{cases} \frac{1}{6} & x = 1, 2, 3, \ldots, 6 \\ 0 & \text{other wise} \end{cases}$$

Random Variable (r.v.)

متقطع	مستمر
discretes (r.v.)	Continuous (r.v.)
$x = 0, 1, 2, 3, 4, \ldots$	$a < x < b$
Probability mass function	Probability density function
دالة كتلة احتمالية	دالة كثافة احتمالية
(p.m.f.) رمزها	(p.d.f.) رمزها
في اغلب الاحيان $P(x)$ شكلها	$f(x)$ شكلها



خواص الدالة



1- $P(x) \geq 0$; $0 \leq P(x_i) \leq 1$, $i = 1, 2, 3, \ldots$	$f(x) \geq 0$; $0 \leq f(x) \leq 1$
2- $\sum_{i=1}^{n} P(x_i) = 1$	$\int_{-\infty}^{\infty} f(x)\, dx = 1$

دالة الاحتمال التراكمية

Cumulative Probability function

$$F(x) = \sum_{x_i < x} P(x_i) \qquad \Big| \qquad F(x) = \int_{-\infty}^{x} f(x)\, dx$$

2– Distribution Function (D.F.) دالة التوزيع

Definitions:

If X is a random variable (r.v.) defined on the sample space S , the cumulative distribution function (c. d. f.) or cumulative probability function (c. p. f.) of a discrete random variable , denoted by $F(x)$ & is defined by :-

$$F(x) = P(X \leq x) \quad \text{for any real number}$$
$$= \sum P(x_i)$$

Properties :

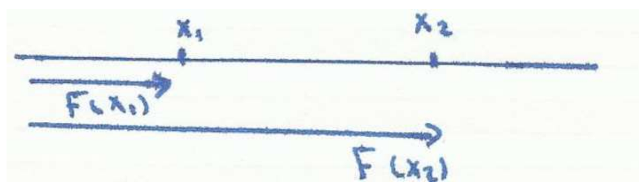
The distribution function $F(x)$ has following properties :-

1_ $F(\infty) = \lim_{x \to \infty} F(x) = 1$

2_ $F(-\infty) = \lim_{x \to -\infty} F(x) = 0$

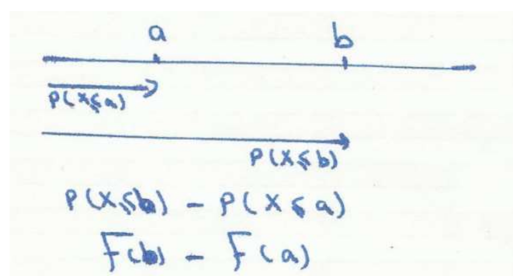
$0 \leq F(x) \leq \mathbf{1}$

3_ $F(x_1) \leq F(x_2)$; $if \;\; x_1 \leq x_2$



4_ $\boldsymbol{P(a < x \leq b)} = F(b) - F(a) \quad for\ all \quad a < b$

$\boldsymbol{P(x > b)} = 1 - F(b)$



Ex)

$$\boldsymbol{P(x)} = \begin{cases} \frac{x}{6} & , \quad \boldsymbol{x = 1, 2, 3} \\ \mathbf{0} & , \quad other\ wise \end{cases}$$

Find :

1_ Probability mass function

2_ cumulative probability function