Then the density functions is:

$$f(z_1) = F'(z_1) = \lambda e^{-\lambda t} \quad , \quad z_1 > 0$$

Is an exponential dist for Z_1 with mean $\frac{1}{\lambda}$.

To obtain the distribution of Z_2 given Z_1

$$P_r\{Z_2 > t \mid Z_1 = S\} = P_r\{0 \text{ events occur in } (s, s+t) \mid Z_1 = s\} \qquad (s<t)$$

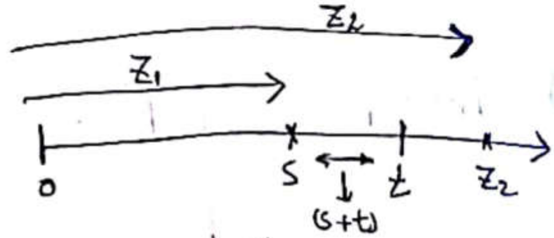
$$= P_r\{0 \text{ events occur in } (s, s+t)\}$$

$$= P_r\{X(t) = 0\} \qquad \text{"independent increment}$$

$$\therefore P_r\{Z_2 > t \mid Z_1 = s\} = e^{-\lambda t} \qquad \text{[by stationary increments]}$$

Therefore Z_2 is also an exponential distribution with mean $\frac{1}{\lambda}$ then repeting the same argument yields to:

For $n = 1, 2, \ldots$, Z_n are independent identically distribution exponential r.v's having mean $\frac{1}{\lambda}$.



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