

Solution

① Optimist criterion

$$\text{Max } a_1 = \text{Max}(3, 5, 8, -1) = 8$$

$$\text{Max } a_2 = \text{Max}(6, 5, 2, 0) = 6$$

$$\text{Max } a_3 = \text{Max}(0, 5, 6, 4) = 6$$

$$\text{Max}(8, 6, 6) = 8$$

$\therefore a_1$ is the best alternative

② pessimist criterion (Wald)

$$\text{Min } a_1 = \text{Min}(3, 5, 8, -1) = -1$$

$$\text{Min } a_2 = \text{Min}(6, 5, 2, 0) = 0$$

$$\text{Min } a_3 = \text{Min}(0, 5, 6, 4) = 0$$

$$\text{Max}(-1, 0, 0) = 0$$

a_2 or a_3 is the best alternative

③ Laplace

$$\text{mean } a_1 = (3 + 5 + 8 + (-1))/4 = 3.75$$

$$\text{mean } a_2 = (6 + 5 + 2 + 0)/4 = 3.25$$

$$\text{mean } a_3 = (0 + 5 + 6 + 4)/4 = 3.75$$

$$\text{Max}(3.75, 3.25, 3.75) = 3.75$$

a_1 or a_3 is the best alternative

④ Hurwicz criterion

$$HV(a_1) = 0.4(8) + (1 - 0.4)(-1) = 2.6$$

$$HV(a_2) = 0.4(6) + 0.6(0) = 2.4$$

$$HV(a_3) = 0.4(6) + 0.6(0) = 2.4$$

$$\text{Max}(2.6, 2.4, 2.4) = 2.6$$

a_1 is the best alternative

⑤ Savage criterion

Alternatives	States of Nature			
	S_1	S_2	S_3	S_4
a_1	$6-3=3$	$5-5=0$	$8-8=0$	$4-(-1)=5$
a_2	$6-6=0$	$5-5=0$	$8-2=6$	$4-0=4$
a_3	$6-0=6$	$5-5=0$	$8-6=2$	$4-4=0$

$$SV(a_1) = \max(3, 0, 0, 5) = 5$$

$$SV(a_2) = \max(0, 0, 6, 4) = 6$$

$$SV(a_3) = \max(6, 0, 2, 0) = 6$$

$$\min(5, 6, 6) = 5$$

a_1 is the best alternative.

⑥ Expected Value of returns

	States of Nature			
	$P(S_1)=0.6$	$P(S_2)=0.1$	$P(S_3)=0.2$	$P(S_4)=0.1$
	S_1	S_2	S_3	S_4
a_1	3	5	8	-1
a_2	6	5	2	0
a_3	0	5	6	4

$$E(a_1) = 0.6(3) + 5(0.1) + 8(0.2) + 1(0.1) = 3.8$$

$$E(a_2) = 6(0.6) + 5(0.1) + 2(0.2) + 0(0.1) = 4.5$$

$$E(a_3) = 0(0.6) + 5(0.1) + 6(0.2) + 4(0.1) = 2.1$$

$$\text{Max}(3.8, 4.5, 2.1) = 4.5$$

a_2 is the best alternative.

⑦ Expected Value Criterion for opportunity less regret

we make the regret table

Alternatives	States of Nature			
	S_1	S_2	S_3	S_4
a_1	3	0	5	5
a_2	0	6	4	4
a_3	6	2	0	0

$$p(S_1) = 0.6$$

$$p(S_2) = 0.1$$

$$p(S_3) = 0.2$$

$$p(S_4) = 0.1$$

$$EL(a_1) = 0.6(3) + 0(0.1) + 5(0.2) + 5(0.1) = 2.3$$

$$EL(a_2) = 0(0.6) + 6(0.1) + 4(0.2) + 4(0.1) = 1.6$$

$$EL(a_3) = 6(0.6) + 2(0.1) + 0(0.2) + 0(0.1) = 4$$

$$EL = \text{Min}(2.3, 1.6, 4) = 1.6$$

a_2 is the best alternative

⑧ The criterion for states of nature is the frequent occurrence

$$p^* = \max(0.6, 0.1, 0.2, 0.1) = 0.6 \Rightarrow S_1$$

$$ML(a_1) = 3, ML(a_2) = 6, ML(a_3) = 0$$

$$ML = \max(3, 6, 0) = 6$$

a_2 is the best alternative.

Example: The following return Matrix is present cost matrix. Find the best alternative using

① optimist

② pessimist

③ Laplace

④ Hurwicz $\alpha = 0.4$

⑤ Savage criterion

⑥ Expected Value Criterion

For opportunity Loss

EOL

$$P(S_1) = 0.4, P(S_2) = 0.3$$

$$P(S_3) = 0.3$$

strategies	States of Nature		
	Refreshing market	Good Market	Stagnat Market
Investing in food trade	5	2	1
Investing in real estate	4	3	3
Investing in stocks	2	6	1

① Optimist criterion

$$\text{Min } a_1 = \text{Min}(5, 2, 1) = 1$$

$$\text{Min}(4, 3, 3) = 3$$

$$\text{Min}(2, 6, 1) = 1$$

$$\text{Min}(1, 3, 1) = 1$$

a_1 or a_3

② Pessimist criterion

$$\text{Max } a_1 = \text{Max}(5, 2, 1) = 5$$

$$\text{Max } a_2 = \text{Max}(4, 3, 3) = 4$$

$$\text{Max } a_3 = \text{Max}(2, 6, 1) = 6$$

$$\text{Min}(5, 4, 6) = 4$$

a_2 is the best alternative

③ Laplace

$$\text{Mean } a_1 = (5 + 2 + 1) / 3 = 8 / 3$$

$$\text{Mean}(4 + 3 + 3) / 3 = 11 / 3$$

$$\text{Mean}(a_3) = \text{mean}(2 + 6 + 1) = 9 / 3$$

$$\text{Min}(2.67, 3.33, 3) = 2.67$$

a_1 is the best alternative

④ Hurwicz $\alpha = 0.4$

$$\text{HV}(a_1) = 0.4(1) + 0.6(5) = 3.4$$

$$\text{HV}(a_2) = 0.4(3) + 0.6(4) = 3.6$$

$$\text{HV}(a_3) = 0.4(1) + 0.6(6) = 4$$

$$\text{Min}(3.4, 3.6, 4) = 3.4$$

a_1 is the best alternative

Savage criterion

	s_1	s_2	s_3
a_1	$5-2=3$	$2-2=0$	$1-1=0$
a_2	$4-2=2$	$3-2=1$	$3-1=2$
a_3	$2-2=0$	$6-2=4$	$1-1=0$

$$SV(a_1) = \max(3, 0, 0) = 3$$

$$SV(a_2) = \max(2, 1, 2) = 2$$

$$SV(a_3) = \max(0, 4, 0) = 4$$

$$\min(3, 2, 4) = 2$$

a_2 is the best

$$\textcircled{6} EOL(a_1) = 3(0.4) + 0 + 0 = 1.2$$

$$EOL(a_2) = 2(0.4) + 1(0.3) + 1(0.3) \\ = 0.8 + 0.6 = 1.4$$

$$EOL(a_3) = 0 + 4(0.3) = 1.2$$

$$\min(1.2, 1.4, 1.2) = 1.2$$

a_1 or a_3

$\textcircled{7}$ Most likelihood stat