1) ان المسافر او الراكب سوف ينتظر اقل من 7 دقائق

$6{:}3 \rightarrow 6{:}10 \quad or \quad 6{:}13 \rightarrow \ 6{:}20$

$P(3 < x < 10) + P(13 < x < 20)$

$\int_3^{10} f(x)dx + \int_{13}^{20} f(x)dx = \int_3^{10} \frac{1}{20}dx + \int_{13}^{20} \frac{1}{20}dx = \frac{1}{20}[(10-3)+(13-20)]$

$\frac{1}{20}[7+7] = \frac{14}{20} = \frac{7}{10} = 0.7$

2) ان المسافر او الراكب سوف ينتظر اكثر من 4 دقائق

$6{:}0 \rightarrow 6{:}4 \quad or \quad 6{:}10 \rightarrow \ 6{:}14$

$P(0 < x < 4) + P(10 < x < 14)$

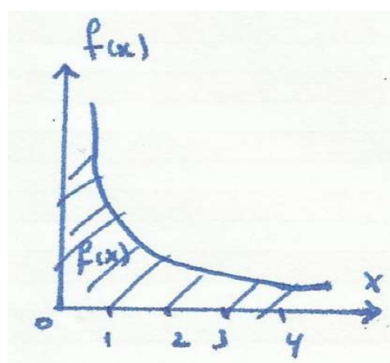
$$\int_0^4 f(x)dx + \int_{10}^{14} f(x)dx = \int_0^4 \frac{1}{20}dx + \int_{10}^{14} \frac{1}{20}dx = \frac{1}{20}[(4-0)+(14-10)]$$

$\frac{1}{20}[4+4] = \frac{8}{20} = \frac{4}{10} = 0.4$

Or $\quad P(0 < x < 4) = F(b) - F(a) = F(4) - F(0)$

3.2) Exponential Distribution:

A r.v. X is said to have an exponential distribution if its *p.d.f.* (probability density function) is given by:



$$f(X) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & 0 \le X \\ 0 & o.w. \end{cases}$$

Or

$$f(X) = \begin{cases} \theta e^{-\theta x} & 0 \le X \\ 0 & o.w. \end{cases}$$

Where $\theta > 0$

Exponential distribution $\quad x \sim Exp(\theta)$

θ : is a parameter distribution.

To show that $f(X)$ is *p.d.f.* then: $\int_{-\infty}^{\infty} f(X)dx = 1$

$$\int_0^\infty \frac{1}{\theta} e^{-\frac{x}{\theta}} dx = \frac{1}{\theta}\int_0^\infty e^{-\frac{x}{\theta}} dx = -(e^{-\frac{x}{\theta}}|_0^\infty = -\left(e^{-\frac{\infty}{\theta}} - e^{-\frac{0}{\theta}}\right) = -(0-1) = 1$$

The distribution function (D.f) or $F(X)$ is given by:

$$F(X) = \int_{-\infty}^{x} f(t)dt$$

Then

$$F(X) = \int_0^x \frac{1}{\theta} e^{-\frac{t}{\theta}} dt = -(e^{-\frac{t}{\theta}}|_0^x = -\left(e^{-\frac{x}{\theta}} - 1\right) = 1 - e^{-\frac{x}{\theta}}$$

$$\therefore F(X) = \begin{cases} 0 & X < 0 \\ 1 - e^{-\frac{x}{\theta}} & 0 \le X \end{cases}$$

Or

$$F(X) = \begin{cases} 0 & X < 0 \\ 1 - e^{-x\theta} & 0 \le X \end{cases}$$

Ex) The mileage (المسافة المقطوعة) (in thousand of miles) which car owners (مالك السيارة) get with a certain kind of tyer is r.v. having an exponential with $\theta = 20$. Find the probability that one of these tyers will last (يتحمل او يدوم):

1) At most 12000 miles

2) Anywhere from 18000 to 24000 miles

Sol)

Let X denote the mileage of this tyer (in 1000 miles)

1) $P(X \le 12) = \int_0^{12} \frac{1}{20} e^{-\frac{x}{20}} dx = -(e^{-\frac{x}{20}}|_0^{12} = -\left(e^{-\frac{12}{20}} - e^{-\frac{0}{20}}\right) = 1 - e^{-0.6}$

$= 1 - 0.549 = 0.451$

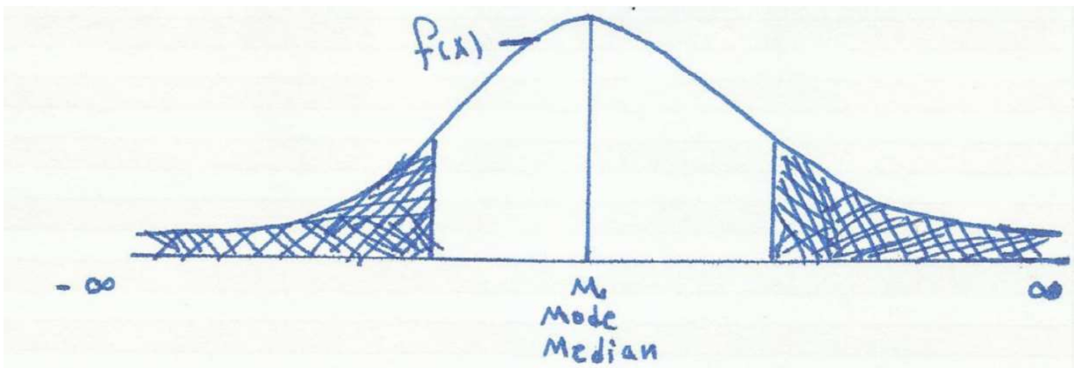
2) $P(18 < X < 24) = \int_{18}^{24} \frac{1}{20} e^{-\frac{x}{20}} dx = -(e^{-\frac{x}{20}}|_{18}^{24} = -\left(e^{-\frac{24}{20}} - e^{-\frac{18}{20}}\right)$

$= -(e^{-1.2} - e^{-0.9}) = -(0.301 - 0.406) = 0.105$

3.3) Normal Distribution:

The r.v. X is said to have an Normal distribution if its *p.d.f.* (probability density function) is given by:

$$f(X) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} & -\infty < X < \infty \\ 0 & o.w. \end{cases}$$



Normal distribution $\quad x \sim N(\mu, \sigma^2)$

μ, σ^2 : are the parameters distribution.

Nots:

1) If $x \sim N(\mu, \sigma^2)$, then $Z = \frac{x-\mu}{\sigma}$ is standard normal variate with

$Mean = \mu = 0 \quad \& \quad variance = \sigma^2 = 1$

$\therefore\ x \sim SN(0,1)$

2) the *P.d.f.* of the standard normal variate Z is given by

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2} \qquad -\infty < Z < \infty$$

And the corresponding distribution function $\emptyset(z) = P(Z \leq z)$ is given by:

$$\emptyset(z) = P(Z \leq z) = \int_{-\infty}^{Z} f(u)du = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

Or

The value of $\emptyset(z)$ for $Z \geq 0$ are given in table of the standard normal distribution.

او يتم الحصول على قيمة $\emptyset(z)$ من الجداول الإحصائية للتوزيع الطبيعي القياسي.

$$P(a < Z < b) = \int_a^b \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2} dz$$

Or

$$P(a < Z < b) = \emptyset(b) - \emptyset(a) \quad for \quad a < b$$

Note:

$$\emptyset(z) = P(Z \leq z)$$

$$P(a < Z < b) = \emptyset(b) - \emptyset(a)$$

$$\therefore P(Z < -a) = P(Z > a) = 1 - P(Z \leq a) = 1 - \emptyset(a) = \emptyset(-a)$$

- $P(2 < Z < 3.1) = \emptyset(3.1) - \emptyset(2) = 0.999 - 0.9772 = 0.0218$
- $P(Z \leq -0.56) = P(Z > 0.56) = 1 - P(Z \leq 0.56)$

  $= 1 - \emptyset(0.56) = 1 - 0.7123 = 0.2877$

If $x \sim N(\mu, \sigma^2)$, the distribution function of X can be expressed as:

$$F(a) = P(X \leq a)$$

$$\therefore F(a) = P(X \leq a) = P(\frac{x-\mu}{\sigma} < \frac{a-\mu}{\sigma})$$

$$= P\left(Z < \frac{a-\mu}{\sigma}\right) = \emptyset(\frac{a-\mu}{\sigma})$$

Ex) $x \sim N(4,25)$

1) $P(X < 6) = P\left(\frac{x-\mu}{\sigma} < \frac{6-\mu}{\sigma}\right) = P\left(Z < \frac{6-4}{5}\right) = P\left(Z < \frac{2}{5}\right) = P(Z < 0.04)$

$= \emptyset(0.4) = 0.554$