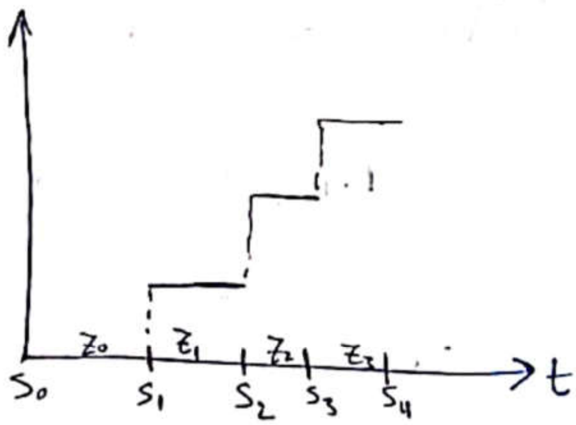
Theorem (3): The waiting time S_n has the gamma distribution whose Probability density function is:

$$f_{S_n}(t) = \frac{\lambda^n \, t^{n-1}}{(n-1)!} e^{-\lambda t} \quad , \; n = 1, 2, \dots, t \geq 0$$

Where the first event S_1 is exponentially distributed

$$f_{S_1} = \lambda e^{-\lambda t} \quad , \; t \geq 0$$

Proof: Let S_n be the arrival time of the nth event. Called waiting time until the nth event occure then $S_n = \sum_{i=1}^{n} Z_i$, $n \geq 1$



ثم توضيح

The event $S_n \leq t$ occur iff there are at least n events in interval $(0,t]$, and Since the number of events in $(0,t]$ has a Poisson distribution with mean (λt), then:

$$F_{S_n}(t) = P_r\{S_n \leq t\}$$

$$= P_r\{X(t) \geq n\}$$