$$\frac{1}{w} = X_0'(X'X)^{-1}X_0 = 6.5$$

So, the 95% confidence interval for the mean of y at that point is:

$$\hat{y}_0 - t_{\frac{1}{2}\alpha,n-m-1}\sqrt{\sigma^2_{\hat{y}_X}} \le y_0 \le \hat{y}_0 + t_{\frac{1}{2}\alpha,n-m-1}\sqrt{\sigma^2_{\hat{y}_X}}$$

$$\hat{y}_0 - t_{\frac{1}{2}\alpha,n-m-1}\sqrt{\frac{MSe}{w}} \le y_0 \le \hat{y}_0 + t_{\frac{1}{2}\alpha,n-m-1}\sqrt{\frac{MSe}{w}}$$

$$\hat{y}_0 - t_{\frac{1}{2}\alpha,n-m-1}\sqrt{MSe\, X_0'(X'X)^{-1}X_0} \le y_0 \le \hat{y}_0 + t_{\frac{1}{2}\alpha,n-m-1}\sqrt{X_0'(X'X)^{-1}X_0}$$

$$18.1665 - (3.182)\sqrt{(1.22)(6.5)} \le y_0 \le 18.1665 + (3.182)\sqrt{(1.22)(6.5)}$$

$$9.205 \le y_0 \le 27.127$$

Additional sum of squares

Assuming we have the following model:

$$y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + e_i$$

The vector of estimated parameters will be as follows:

$$\hat{\beta} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \\ \hat{\beta}_5 \end{bmatrix}$$

What is meant by additional sum of squares?

What is meant by the additional sum of squares is the amount of squares that a particular variable adds as a result of adding it to existing variables in the model. In many cases, the researcher wants to know whether there is a partial set of independent variables (let it be X4 and X5, for example) that have a significant effect on the prediction of Y for a model that contains the variables X1, X2, and X3.

The sum of squares for such a case is represented by the following formula:

$$SSR(X_4, X_5 / X_1, X_2, X_3)$$

There are two methods for finding the additional sum of squares:

1- **The usual method**

This method is represented by using the total sum of squares minus the sum of squares for the model containing the fixed variables.

To find the additional sum of squares for adding variables X5 and X4 to a model containing variables X3, X2 and X1, we use the following formula:

$SSR(X_4, X_5/X_1, X_2, X_3) = SSR(X_1, X_2, X_3, X_4, X_5) - SSR(X_1, X_2, X_3)$

Whereas $SSR(X_1, X_2, X_3, X_4, X_5)$ It is a model with five variables.

and that $SSR(X_1, X_2, X_3)$ It is a three-variable model.

2- Using the shortcut method

The short method is represented by the following formula:

$$SR(X_4, X_5/X_1, X_2, X_3) = [\hat{\beta}_4 \quad \hat{\beta}_5] \begin{bmatrix} C_{44} & C_{45} \\ C_{54} & C_{55} \end{bmatrix}^{-1} \begin{bmatrix} \hat{\beta}_4 \\ \hat{\beta}_5 \end{bmatrix}$$

Where the values $\hat{\beta}_4$ و $\hat{\beta}_5$ It is taken from the overall model, as well as the matrix $\begin{bmatrix} C_{44} & C_{45} \\ C_{54} & C_{55} \end{bmatrix}$ Taken from the matrix $(X'X)^{-1}$ After deleting all C values except the values of the estimated parameters $\hat{\beta}_4$ و $\hat{\beta}_5$.

Another example: To find the additional sum of squares for one independent variable, let it be X3, added to a model containing the variables X5, X4, X2, and X1, we use the following formula:

$SSR(X_3/X_1, X_2, X_4, X_5) = SSR(X_1, X_2, X_3, X_4, X_5) - SSR(X_1, X_2, X_4, X_5)$

Where is the model $SSR(X_1, X_2, X_3, X_4, X_5)$ It is a model that contains all variables X_1, X_2, X_3, X_4, and X_5.

And that model $SSR(X_1, X_2, X_4, X_5)$ It is a model that contains only four variables X_1, X_2, X_4, X_5.

Or we can find the additional sum of squares $SSR(X_3/X_1, X_2, X_4, X_5)$ In short, by:

$$SSR(X_3/X_1, X_2, X_4, X_5) = \hat{\beta}_3{}' C_{33}^{-1} \hat{\beta}_3 = \frac{\hat{\beta}_3{}^2}{C_{33}}$$

Example: If you have the following data:

X4	X3	X2	X1	y
1	10	6	4	3
4	8	7	5	6
2	2	8	3	8
5	4	3	4	9
4	6	2	3	4
3	3	1	5	2

$$(X'X)^{-1} = \begin{bmatrix} 6.836 & -0.9723 & -0.2434 & -0.0517 & -0.442 \\ -0.9723 & 0.2811 & 0.0135 & -0.0234 & -0.0261 \\ -0.2434 & 0.0135 & 0.0314 & -0.005 & 0.0237 \\ -0.0517 & -0.0234 & -0.005 & 0.0248 & 0.0103 \\ -0.442 & -0.0261 & 0.0237 & 0.0103 & 0.1208 \end{bmatrix}$$

Required: Find the additional sum of squares to add the variables X2 and X1 to a model containing X4, X3 using the normal and short method.

Solution:

We first estimate a total parameter model, i.e. the model that containing all the parameters:

$y_i = 2.08 - 0.74\, X_1 + 0.768\, X_2 - 0.299\, X_3 + 1.39\, X_4$

<u>Usual method:</u>

$$SSR(X_1, X_2/X_3, X_4) = SSR(X_1, X_2, X_3, X_4) - SSR(\, X_3, X_4)$$

First, we find the value of the sum of squares for a model containing all variables X1, X2, X3, and X4, as follows:

$$SSR(X_1, X_2, X_3, X_4) = \hat{\beta}' X' y - n\bar{y}^2$$