Ex: Use Gramer's method to solve the following Linear system.

$$2x_1 + 3x_2 + x_3 = 1$$
$$-2x_2 + x_3 = 0$$
$$x_1 + x_2 + x_3 = 2$$

Sol:

$m = n = 3$,

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 & 1 \\ 0 & -2 & 1 \\ 1 & 1 & 1 \end{vmatrix}\begin{matrix} 2 & 3 \\ 0 & -2 \\ 1 & 1 \end{matrix} = (-4+3+0)-(-2+2+0) = -1$$

$\therefore |A| \neq 0$

$$|A_1| = \begin{vmatrix} 1 & 3 & 1 \\ 0 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix}\begin{matrix} 1 & 3 \\ 0 & -2 \\ 2 & 1 \end{matrix} = (-2+6+0)-(-4+1+0) = 7$$

$$|A_2| = \begin{vmatrix} 2 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 2 & 1 \end{vmatrix}\begin{matrix} 2 & 1 \\ 0 & 0 \\ 1 & 2 \end{matrix} = -3$$

$$|A_3| = -6$$

$$\therefore x_1 = \frac{|A_1|}{|A|} = \frac{7}{-1} = -7$$

$$x_2 = \frac{|A_2|}{|A|} = \frac{-3}{-1} = 3$$

$$x_3 = \frac{|A_3|}{|A|} = \frac{-6}{-1} = 6$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 6 \end{bmatrix}$$

Ex(3) جد الحل المناسب للنظام الخطي التالي
باستخدام طريقة كرامر

$$x_1 - 3x_2 = 5$$
$$-x_2 + x_3 = -1$$
$$6x_1 + 2x_3 = 0$$

Sol:

$$A = \begin{bmatrix} 1 & -3 & 0 \\ 0 & -1 & 1 \\ 6 & 0 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix}$$

$$|A| = \left| \begin{matrix} 1 & -3 & 0 \\ 0 & -1 & 1 \\ 6 & 0 & 2 \end{matrix} \right| \begin{matrix} 1 & -3 \\ 0 & -1 \\ 6 & 0 \end{matrix}$$

$= (-2 - 18 + 0) - (0 + 0 + 0) = -20 \neq$ صفر

∴ يوجد حل للنظام

$$x_1 = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} 5 & -3 & 0 \\ -1 & -1 & 1 \\ 0 & 0 & 2 \end{vmatrix}}{-20} = \frac{-16}{-20} = \frac{4}{5}$$

$$x_2 = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} 1 & 5 & 0 \\ 0 & -1 & 1 \\ 6 & 0 & 2 \end{vmatrix}}{-20} = \frac{28}{-20} = \frac{-7}{5}$$

$$x_3 = \frac{|A_3|}{|A|} = \frac{\begin{vmatrix} 1 & -3 & 5 \\ 0 & -1 & 1 \\ 6 & 0 & 0 \end{vmatrix}}{-20} = \frac{48}{-20} = \frac{-12}{5}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4/5 \\ -7/5 \\ -12/5 \end{bmatrix}$$

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matrix method

٢- طريقة المصفوفات

وهي تستخدم في حالة النظام الخطي الغير متجانس $(AX=B)$ وعندما $m=n$, $|A| \neq 0$

اشتقاق القانون

$$AX = B$$

$$A^{-1} \cdot A X = A^{-1} B$$

$$I \cdot X = A^{-1} B$$

$$\boxed{\therefore X = A^{-1} B}$$ القانون العام

Ex: Find the solution to the following system using the matrix method

جد الحل للنظام الخطي باستخدام طريقة المصفوفات

$$3X_1 + 4X_2 = 5$$
$$2X_1 + 3X_2 = 3$$

sol: $m = n = 2$, $|A| = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} = 9 - 8 = 1 \neq 0$

$\therefore$ يوجد حل وحيد

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$X = A^{-1} B = \frac{adj(A)}{|A|} \cdot B = \frac{\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}}{1} \cdot \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$= \underset{2\times 2}{\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 5 \\ 3 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} 3 \\ -1 \end{bmatrix}}$$

(2)

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

التحقيق

$$3(3) + 4(-1) = \underline{5}$$
$$9 - 4 = \underline{5}$$

Ex(2):

$$3x_1 - 2x_2 = 4$$
$$-x_1 + 4x_2 = 7$$

$m = n$

$|A| \neq 0$

$$X = A^{-1}B \qquad A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}, \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

$$A^{-1} = \frac{adj(A)}{|A|} = \cancel{\frac{\begin{vmatrix} 3 & -2 \\ -1 & 4 \end{vmatrix}}{10} = \frac{1}{10}\left[\ \right]}$$

$$A^{-1} = \frac{\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix}}{10} = \frac{1}{10}\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix}$$

$$X = A^{-1}B = \frac{1}{10}\underbrace{\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}}_{2\times 2}\underbrace{\begin{bmatrix} 4 \\ 7 \end{bmatrix}}_{2\times 1} = \frac{1}{10}\begin{bmatrix} 30 \\ 25 \end{bmatrix} = \begin{bmatrix} 3 \\ 2.5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2.5 \end{bmatrix}$$

التحقق

$$3(3) - 2(2.5) = 4$$
$$3 + \underbrace{4(2.5)}_{10.0} = \underline{7}$$

Ex2 Find the solution to the following system using the matrix method

$$4x_1 + 3x_2 + 2x_3 = 11$$
$$3x_1 + x_2 - 2x_3 = -1$$
$$-x_1 + 3x_3 = 7$$

Sol.: $\underline{m = n = 3}$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 1 & -2 \\ -1 & 0 & 3 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad B = \begin{bmatrix} 11 \\ -1 \\ 7 \end{bmatrix}$$

$$X = A^{-1} B$$

$$A^{-1} = \frac{adj(A)}{|A|}$$

$$|A| = \left| \begin{matrix} 4 & 3 & 2 \\ 3 & 1 & -2 \\ -1 & 0 & 3 \end{matrix} \right| \begin{matrix} 4 & 3 \\ 3 & 1 \\ -1 & 0 \end{matrix} = (12 + 6 + 0) - (-2 + 0 + 27)$$
$$= 18 - 25 = -7 \neq 0$$

$\alpha_{11} = 3$ $\qquad \alpha_{21} = -7$ $\qquad \alpha_{31} = 1$

$\alpha_{12} = 7$ $\qquad \alpha_{22} = 14$ $\qquad \alpha_{32} = -3$

$\alpha_{13} = -8$ $\qquad \alpha_{23} = 14$ $\qquad \alpha_{33} = -5$

$$\therefore cof(A) = \begin{bmatrix} 3 & -7 & 1 \\ -9 & 14 & -3 \\ -8 & 14 & -5 \end{bmatrix} \longrightarrow adj(A) = \begin{bmatrix} 3 & -9 & -8 \\ -7 & 14 & 14 \\ 1 & -3 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{adj(A)}{|A|} = -1/7 \begin{bmatrix} 3 & -9 & 8 \\ -7 & 14 & 14 \\ 1 & -3 & -5 \end{bmatrix}$$

$\therefore X = A^{-1} B$

$$= -\frac{1}{7} \begin{bmatrix} 3 & -9 & -8 \\ -7 & 14 & 14 \\ 1 & -3 & -5 \end{bmatrix}_{3\times 3} \begin{bmatrix} 11 \\ 1 \\ 7 \end{bmatrix}_{3\times 1}$$

$$\therefore X = -\frac{1}{7} \begin{bmatrix} -14 \\ 7 \\ -21 \end{bmatrix} \longrightarrow X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

Ex:

$2x_1 + 3x_2 = 2$

$3x_1 + 4x_2 = 5$

Sol: $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}, \; X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \; B = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$X = A^{-1} B$

$$A^{-1} = \frac{\operatorname{adj}(A)}{|A|} = \frac{\begin{vmatrix} 4 & -3 \\ -3 & 2 \end{vmatrix}}{-1} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}_{2\times 2} \begin{bmatrix} 2 \\ 5 \end{bmatrix}_{2\times 1} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$$