

$$y_m = y_0 + \delta y_{1/2} + \frac{m(m-1)}{2!} \delta^2 y_1 + \frac{m(m-1)(m-2)}{3!} \delta^3 y_{3/2} + \dots$$

$$\begin{aligned} y_0 &= \frac{1}{2} y_0 + \frac{1}{2} y_0 + \frac{1}{2} y_1 - \frac{1}{2} y_1 \\ &= \frac{1}{2} (y_0 + y_1) - \frac{1}{2} (y_1 - y_0) \\ &= \frac{1}{2} (y_0 + y_1) - \frac{1}{2} \delta y_{1/2} \dots \dots \dots (1) \end{aligned}$$

$$\begin{aligned} \delta^2 y_1 &= \frac{1}{2} \delta^2 y_0 - \frac{1}{2} \delta^2 y_0 + \frac{1}{2} \delta^2 y_1 + \frac{1}{2} \delta^2 y_1 \\ &= \frac{1}{2} \delta^2 (y_0 + y_1) + \frac{1}{2} \delta^2 (y_1 - y_0) \\ &= \frac{1}{2} \delta^2 (y_0 + y_1) + \frac{1}{2} \delta^3 y_{1/2} \dots \dots \dots (2) \end{aligned}$$

$$\begin{aligned} \delta^3 y_{3/2} &= \delta^3 y_{3/2} - \delta^3 y_{1/2} + \delta^3 y_{1/2} \\ &= \delta^3 (y_{3/2} - y_{1/2}) + \delta^3 y_{1/2} \\ &= \delta^3 \delta y_1 + \delta^3 y_{1/2} \\ &= \frac{1}{2} \delta^4 (y_0 + y_1) + \frac{1}{2} \delta^5 y_{1/2} + \delta^3 y_{1/2} \dots \dots \dots (3) \end{aligned}$$

نعرف المؤثر الاتي:

$$\mu y_{i+1/2} = \frac{1}{2} (y_i + y_{i+1})$$

ويسمى مؤثر المعدل

$$\mu y_{1/2} = \frac{1}{2} (y_0 + y_1)$$

$$\therefore y_0 = \mu y_{1/2} - \frac{1}{2} \delta y_{1/2}$$

$$\delta^2 y_1 = \frac{1}{2} \mu \delta^2 y_{1/2} + \frac{1}{2} \delta^3 y_{1/2}$$

$$\delta^3 y_{3/2} = \frac{1}{2} \mu \delta^4 y_{1/2} + \delta^3 y_{1/2} + \frac{1}{2} \delta^5 y_{1/2}$$

$$\begin{aligned}\therefore y_m &= \mu y_{1/2} + \left(m - \frac{1}{2}\right) \delta y_{1/2} + \frac{m(m-1)}{2!} \mu \delta^2 y_{1/2} \\ &+ \frac{m(m-1/2)(m-2)}{3!} \delta^3 y_{3/2} \\ &+ \frac{m(m-1)(m^2-1)(m-2)}{4!} \mu \delta^4 y_{1/2} + \dots\end{aligned}$$

تسمى هذه الصيغة بطريقة الفروقات المركزية للتعديل الداخلي وتسمى أيضا بطريقة ببسل.

مثال:- اكتب جدول الفروقات ومن ثم خمن قيمة الدالة عند النقطة $f(32)$

الحل:-

x_i	y_i	δ	δ^2	δ^3	δ^4
10	0.9848	-0.451	-0.0286	0.0023	0.0007
20	0.9397	-0.0737	-0.0263	0.003	0.0009
30	0.866	-0.1	-0.0233	0.0039	
40	0.766	-0.12	-0.0194		
50	0.6427				

$$m = 0.2, \quad x_m = 32, \quad x_0 = 30$$

$$\mu y_{1/2} = \frac{1}{2} (y_0 + y_1) = 0.816$$

$$\delta y_{1/2} = -0.1 = (y_1 - y_0)$$

$$\mu \delta^2 y_{1/2} = \frac{1}{2} (\delta^2 y_0 + \delta^2 y_1) = -0.0248$$

$$\delta^3 y_{3/2} = 0.003$$

$$\begin{aligned}\cos(32) &= \mu y_{1/2} + \left(m - \frac{1}{2}\right) \delta y_{1/2} + \frac{m(m-1)}{2!} \mu \delta^2 y_{1/2} + \frac{m(m-1/2)(m-2)}{3!} \delta^3 y_{3/2} \\ &= 0.816 + (-0.3)(-0.1) + \frac{0.2(-0.8)}{2!} (-0.0248) + \frac{0.2(-0.3)(-0.8)}{3!} = 0.8482\end{aligned}$$

7- الفروقات المنتهية النسبية (Divide Finite Differences):

تعد هذه الطريقة من الطرق التي تتعامل مع النقاط غير متساوية الابعاد. يعرف الفرق النسبي من الرتبة الاولى كما يلي:

$$\Delta y_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}, i = 0, 1, 2, \dots, n - 1$$

وذو المرتبة الثانية

$$\Delta^2 y_i = \frac{\Delta y_{i+1} - \Delta y_i}{x_{i+2} - x_i}, i = 0, 1, 2, \dots, n - 2$$

وذو الرتبة k

$$\Delta^k y_i = \frac{\Delta^{k-1} y_{i+1} - \Delta^{k-1} y_i}{x_{i+k} - x_i}, i = 0, 1, 2, \dots, (n - k)$$

x_i	y_i	Δ	Δ^2	Δ^3
x_0	y_0	$\Delta y_0 = \frac{y_1 - y_0}{x_1 - x_0}$	$\Delta^2 y_0 = \frac{\Delta y_1 - \Delta y_0}{x_2 - x_0}$	$\Delta^3 y_0 = \frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0}$
x_1	y_1	$\Delta y_1 = \frac{y_2 - y_1}{x_2 - x_1}$	$\Delta^2 y_1 = \frac{\Delta y_2 - \Delta y_1}{x_3 - x_1}$	$\Delta^3 y_1 = \frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1}$
x_2	y_2	$\Delta y_2 = \frac{y_3 - y_2}{x_3 - x_2}$	$\Delta^2 y_2 = \frac{\Delta y_3 - \Delta y_2}{x_4 - x_2}$	
x_3	y_3	$\Delta y_3 = \frac{y_4 - y_3}{x_4 - x_3}$		
x_4	y_4			

الصيغة العامة للفروقات النسبية للتعديل الداخلي يمكن كتابتها كل التالي:

$$y_m = y_0 + (x_m - x_0) \Delta y_0 + (x_m - x_0)(x_m - x_1) \Delta^2 y_0 + (x_m - x_0)(x_m - x_1)(x_m - x_2) \Delta^3 y_0 + \dots$$

مثال:- اكتب جدول الفروقات ومن ثم خمن قيمة الدالة عند النقطة $f(0.5), f(5)$ اذا علمت ان $y = x^3$
لقيم $x = 0, 1, 3, 4$

الحل:-

x_i	y_i	Δ	Δ^2	Δ^3
0	0	1	4	1
1	1	13	8	
3	27	37		
4	64			

$$f(x_m) = y_0 + (x_m - x_0) \Delta y_0 + (x_m - x_0)(x_m - x_1) \Delta^2 y_0 \\ + (x_m - x_0)(x_m - x_1)(x_m - x_2) \Delta^3 y_0$$

$$f(0.5) = 0 + (0.5 - 0)(1) + (0.5 - 0)(0.5 - 1)(4) + (0.5 - 0)(0.5 - 1) \\ (0.5 - 3)(1) = 0.125$$

$$f(5) = 0 + (5 - 0)(1) + (5 - 0)(5 - 1)(4) + (5 - 0)(5 - 1) \\ (5 - 3)(1) = 125$$

مثال:- اكتب جدول الفروقات ومن ثم خمن قيمة الدالة عند النقطة $f(2), f(-1)$ اذا علمت ان
 $y = x^3 + x - 1$ لقيم $x = -2, 0, 1, 4, 5$

الحل:-

x_i	y_i	Δ	Δ^2	Δ^3
-2	-11	5	-1	1
0	-1	2	5	1
1	1	22	10	
4	67	62		
5	129			

$$f(x_m) = y_0 + (x_m - x_0) \Delta y_0 + (x_m - x_0)(x_m - x_1) \Delta^2 y_0 \\ + (x_m - x_0)(x_m - x_1)(x_m - x_2) \Delta^3 y_0$$

$$f(x) = -11 + (x + 2)(5) + (x + 2)(x - 0)(-1) + (x + 2)(x - 0)(x - 1)(1)$$