2) $P(3 < X < 5) = P\left(\frac{3-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{5-\mu}{\sigma}\right)$

$$= P\left(\frac{3-4}{5} < Z < \frac{5-4}{5}\right) = P(-0.2 < Z < 0.2)$$

$$= \emptyset(0.2) - \emptyset(-0.2)$$

$$= \emptyset(0.2) - [1 - \emptyset(0.2)]$$

$$= 0.5793 - [1 - 0.5793] = 0.1586$$

Ex) $x \sim N(3,9)$

1) $P(X > 0) = 1 - P(X \leq 0) = 1 - P(\frac{x-\mu}{\sigma} < \frac{0-\mu}{\sigma})$

$$= 1 - P\left(Z < \frac{0-3}{3}\right) = 1 - P(Z < -1) = 1 - \emptyset(-1)$$

$$= 1 - [1 - \emptyset(1)] = 1 - 1 + \emptyset(1) = \emptyset(1) = 0.84134$$

2) $P(|X-3| > 6) = 1 - P(|X-3| \leq 6) = 1 - P(-6 < X - 3 < 6)$

$$= 1 - P(-6+3 < X - 3 + 3 < 6 + 3)$$

$$= 1 - P(-3 < X < 9) = 1 - P\left(\frac{-3-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{9-\mu}{\sigma}\right)$$

$$= 1 - P\left(\frac{-3-3}{3} < Z < \frac{9-3}{3}\right) = 1 - P(-2 < Z < 2)$$

$$= 1 - [\emptyset(2) - \emptyset(-2)] = 1 - [\emptyset(2) - (1 - \emptyset(2))]$$

Ex) $x \sim N(25,36)$;

$P(|X - 25| \leq c) = 0.9544$; Find the value of c.

Sol)

$P(-c \leq X - 25 \leq c) = 0.9544 \rightarrow P(-c + 25 \leq X \leq c + 25) = 0.9544$

$\rightarrow P\left(\frac{(-c+25)-\mu}{\sigma} \leq \frac{x-\mu}{\sigma} \leq \frac{(c+25)-\mu}{\sigma}\right) = 0.9544$

$\rightarrow P\left(\frac{(-c+25)-25}{6} \leq Z \leq \frac{(c+25)-25}{6}\right) = 0.9544$

$$\rightarrow \quad P\left(\frac{-c}{6} \leq Z \leq \frac{c}{6}\right) = 0.9544$$

$$\because P(a < X < b) = \emptyset(b) - \emptyset(a)$$

$$\therefore \emptyset\left(\frac{c}{6}\right) - \emptyset\left(\frac{-c}{6}\right) = 0.9544 \quad \rightarrow \quad \emptyset\left(\frac{c}{6}\right) - \left[1 - \emptyset\left(\frac{c}{6}\right)\right] = 0.9544$$

$$\rightarrow 2\emptyset\left(\frac{c}{6}\right) - 1 = 0.9544 \quad \rightarrow 2\emptyset\left(\frac{c}{6}\right) = 1 + 0.9544 \ \rightarrow \emptyset\left(\frac{c}{6}\right) = \frac{1.9544}{2}$$

$$\rightarrow \emptyset\left(\frac{c}{6}\right) = 0.9772 = \emptyset(2)$$

$$\rightarrow \emptyset\left(\frac{c}{6}\right) = \emptyset(2) \quad \rightarrow \frac{c}{6} = 2 \quad \Longrightarrow c = 12$$

ملاحظة:

يمكن تحويل توزيع ثنائي الحدين الى التوزيع الطبيعي ايضاً كما هو الحال في توزيع بواسون المتقطع لإيجاد المعدل (λ) وذلك عندما

$$n \rightarrow \infty \qquad \& \qquad P \rightarrow 0$$

ويمكن الحصول على معلمات التوزيع الطبيعي عند توفر توزيع ثنائي الحدين بالمعلمات n, P

$$\{\mu = n\,P \quad ; \quad \sigma^2 = nPq\,\}$$

$$x \sim B(n, P) \xrightarrow[P \rightarrow 0]{n \rightarrow \infty} x \sim N(\,\mu = n\,P,\ \sigma^2 = nPq\,)$$

Where

$$Z = \frac{x - nP}{\sqrt{nPq}}$$

Ex) A fair coin is tossed 20 times. Determine the probability that the number of heads are:

1- 14

2- Between 14 and 16 inclusive, by using:

I) Binomial distribution.

II) Normal approximation to the Binomial distribution.

Sol)

I) $x \sim B\left(20\,,\frac{1}{2}\right)$

1- $P(X = 14) = \ C_{14}^{20}\ \ (\frac{1}{2})^{14}(\frac{1}{2})^{6} = 0.3696$

2- $P(14 \le X \le 16) = P(X = 14) + P(X = 15) + P(X = 16)$

$$= \ C_{14}^{20}\ \ (\frac{1}{2})^{14}(\frac{1}{2})^{6} + C_{15}^{20}\ \ (\frac{1}{2})^{15}(\frac{1}{2})^{5} + C_{16}^{20}\ \ (\frac{1}{2})^{16}(\frac{1}{2})^{4}$$

$$= 0.0563$$

II) $x \sim N(\ \mu = n\ P,\ \ \sigma^2 = nPq\ )$

$x \sim N(\ 20 * \frac{1}{2},\ \ 20 * \frac{1}{2} * \frac{1}{2}\ )$

$x \sim N(\ \mu = 10,\ \ \sigma^2 = 5\ )$

1- $P(X = 14) = P(13.5 \le X \le 14.5)$

$$= P\left(\frac{13.5-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{14.5-\mu}{\sigma}\right)$$

$$= P\left(\frac{13.5-10}{\sqrt{5}} < Z < \frac{14.5-10}{\sqrt{5}}\right)$$

$$= P(1.57 < Z < 2.01)$$

$$= \emptyset(2.01) - \ \emptyset(1.57) = 0.032$$

2- $P(14 \le X \le 16) = P(13.5 \le X \le 16.5)$

$$= P\left(\frac{13.5-10}{\sqrt{5}} < Z < \frac{16.5-10}{\sqrt{5}}\right)$$

$$= P(1.55 < Z < 2.9)$$

$$= \emptyset(2.9) - \ \emptyset(1.55) = 0.0564$$

Generating Function

الدالة المولدة

4.1) Mathematical Expectation: التوقع الرياضي

Expectation of a Discrete Random Variable

* Definition:

The Expectation of a discrete random variable X having a *p.m.f. P(X)* , is denoted be:

$$E(X) = \sum X_i \ P(X_i)$$

Properties:

1- If X is a discrete r.v. with *p.m.f. P(X)* ; then for any real – valued function

$$Y = g(X)$$

$$E(Y) = E\big(g(X)\big) = \sum g(X) \ P(X_i)$$

2- $E(c) = c$

3- $E(cx) = cE(x)$

4- $E(x \mp c) = E(x) \mp c$

5- $E(cx \mp by) = cE(x) \mp bE(y)$

ملاحظة:

يمكن ان تأخذ قيمة التوقع قيمة موجبة او سالبة $-\infty < E(X) < \infty$

Expectation of a Continuous Random Variable

* Definition:

The Expectation of a continuous random variable with *p.d.f. f(x)* , is denoted be:

$$E(X) = \int_{-\infty}^{\infty} x \ f(x) \, dx$$

Let *X* be a r.v. with *p.d.f. f(x)* in the interval [a,b], then:

$$E(X) = E\big(g(X)\big) = \int_a^b E\big(g(X)\big) \, f(x) \, dx$$