3.7 Stationary Distribution or steady state dist. Of M.C

Consider an irreducible positive recurrent and non-periodic M.C (i.e ergodic chain). The probability dist $[\pi_j]$ is called Stationary Distribution of this chain if the system of the linear equations:

$$\pi_j = \sum_{i \in S} \pi_i p_{ij} \quad , j \in S \qquad \dots (1)$$

$$\sum_{j \in S} \pi_j = 1 \qquad \dots (2)$$

Has a solution $\pi = (\pi_1, \pi_2, \dots.)$, if there exists a solutions and we have $\pi_j = \lim_{n \to \infty} p_{ij}^{(n)}$.

Let S={1,2,3}, then the equation 1 and 2 be as:

$\pi_j = \sum_{i=1}^{3} \pi_i p_{ij}$ and $\pi = (\pi_1, \pi_2, \pi_3)$; P= $\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$

Then :

$$\pi_1 = \pi_1 P_{11} + \pi_2 P_{21} + \pi_3 P_{31} \qquad \dots (1)$$

$$\pi_2 = \pi_1 P_{12} + \pi_2 P_{22} + \pi_3 P_{32} \qquad \dots (2)$$

$$\pi_3 = \pi_1 P_{13} + \pi_2 P_{23} + \pi_3 P_{33} \qquad \dots (3)$$

We can Write the equations a above in matrix form as:

$$\pi = \pi P$$

$$\sum \pi_j = 1$$

Example(1): Let $\{X_n, n \geq 0\}$ be a M.C with S={0,1,2} and it's transition matrix as following:

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{matrix}$$

Is this chain has a stationary dist. ? If it has then find it.

Example(2): Let $\{X_n, n \geq 0\}$ be a M.C with S={0,1,2} and it's transition matrix as following:

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.6 & 0 & 0.4 \\ 0 & 0.4 & 0.6 \end{bmatrix} \end{matrix}$$

Is this chain has a stationary dist. ? If it has then find it.