removed), then conduct a regression analysis for each section to find $MSe_i = S_i^2$ for each section, then use the statistical laboratory:

$$F = \frac{Max\ S_i^2}{Min\ S_i^2}$$

If the calculated F value is greater than the table F value, this indicates a difference in variance.

Example: The following table represents data consisting of two variables, the first **of** which represents the average monthly sales of a certain product, represented by (y) in each of the 30 stores, and the amount of monthly advertising expenses on it, represented by (X). Test whether the error variance is constant and homogeneous.

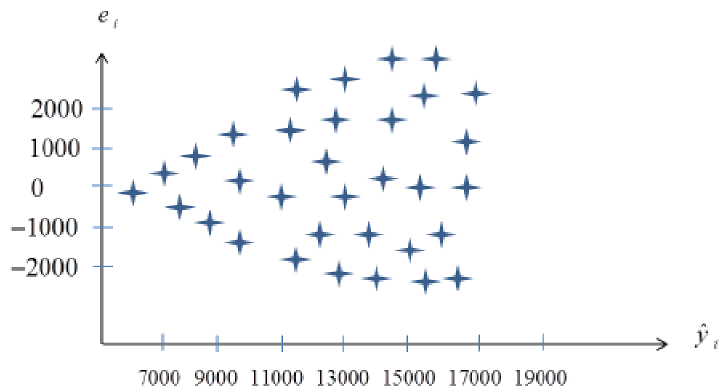
رقم المتجر	X_i	y_i	رقم المتجر	X_i	y_i
1	3000	81464	16	12525	130963
2	3085	72344	17	13700	147041
3	3150	72661	18	14995	180732
4	3225	90743	19	15000	172021
5	5350	98588	20	15050	178187
6	6090	96507	21	15150	155931
7	8885	115814	22	15175	166200
8	8925	126574	23	15200	185304
9	8950	123181	24	16500	188851
10	9000	131434	25	16800	172577
11	9015	114133	26	17830	192424
12	11345	140564	27	19000	218715
13	12275	151352	28	19200	192482
14	12310	144630	29	19350	214317
15	12400	146926	30	19500	203112

The regression line equation and ANOVA table are constructed as follows:
$\hat{y}_i = 4944.383 + 8.0484\, X_{i1}$

S.O.V	d.f	S.S	M.S	$F_{cal.}$
$R(X_i)$	1	4920001188	4920001188	607.46
Error	28	226780353	8099298.3	
Total	29			

To find out the extent to which the conditions of the analysis assumptions (homogeneity of error variance) are met, we follow the following:

1- Using the graph: When plotting e_i against the values of $\hat{y}_i$, the results were as follows:



From the graph it is clear that there is a difference in the error variance.

1- Using a statistical Test, we arrange the data in ascending order according to the independent variable X (which is already arranged in this order), then the data is divided into two parts after deleting the six middle observations (numbered from 13-18) as follows:

رقم المتجر	X_i	y_i	رقم المتجر	X_i	y_i
1	3000	81464	16	12525	130963
2	3085	72344	17	13700	147041
3	3150	72661	18	14995	180732
4	3225	90743	19	15000	172021
5	5350	98588	20	15050	178187
6	6090	96507	21	15150	155931
7	8885	115814	22	15175	166200
8	8925	126574	23	15200	185304
9	8950	123181	24	16500	188851
10	9000	131434	25	16800	172577
11	9015	114133	26	17830	192424
12	11345	140564	27	19000	218715
13	12275	151352	28	19200	192482
14	12310	144630	29	19350	214317
15	12400	146926	30	19500	203112

The following were calculated:

The mean square error for the data of the first section was:

$$MSe_1 = 31066726.3$$

The mean square error for the data of the second section was:

$$MSe_2 = 140450776.4$$

Using the F-test we get:

$$F = \frac{Max\ \ S_i^2}{Min\ \ S_i^2} = \frac{31066726.3}{140450776.4} = 4.52$$

With a significance level of α = 0.05 and degrees of freedom calculated using the following formula:

$$v_1 = v_2 = \frac{n - d - 2k}{2} = 10$$

Where:

n: Sample size which is 30 in our example.

k: Number of estimated parameters which is 2 in our example.

d: Number of deleted intermediate observations which is 6 in our example.

From the F tables we find the tabular value which is: F (0.05,10,10) =3.02

Since the calculated F is greater than the table F, S_1^2 and S_2^2 are different, i.e. there is no homogeneity in the error variance.

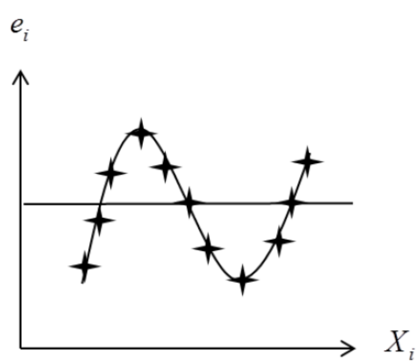
Test whether the errors are independent or if there is an autocorrelation between them

In many problems, especially economic ones, they are in the form of time series, which leads to the error in a specific time period (e_i) being correlated with the error in the time period before it (e_{i-1}) Errors that are correlated over time are called autocorrelation errors.

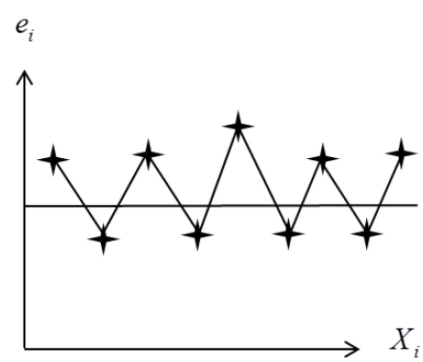
To detect the presence of an autocorrelation between errors, we use the following:

$1-$ Using the graph

When plotting the values of (e_i) against the values of (X_i) in the case of an autocorrelation between the errors, the graph will be in one of the following two forms:



Positive autocorrelation



Negative autocorrelation

In the case of positive autocorrelation, there are several points that are positive, followed by several negative points, followed by several positive points, and so on. In the case of negative autocorrelation, the error points follow one another in sign. For example, the first is positive, the second is negative, the third is positive, the fourth is negative, and so on.

2- Using a statistical laboratory

The Durbin-Watson test can be performed to test whether there is an autocorrelation between errors or not.

To conduct this test, the following hypothesis is tested:

$H_0: r = 0$

$H_1: r \neq 0$

Interpretation of the null hypothesis: There is no autocorrelation between errors

Interpretation of the alternative hypothesis: There is autocorrelation between errors

The Durbin-Watson test formula takes the following form:

$$D = \frac{\sum_{i=2}^{n}(e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i^2}$$

If $D < dl$, we reject the null hypothesis.

If $D > du$ we accept the null.

If $dl \leq D \leq du$, the test is inconclusive.

Where du is the upper limit and dl is the lower limit.

Where the values of du and dl are extracted from Durbin-Watson tables with degrees of freedom n and significance level α, and the number of parameters in the model.

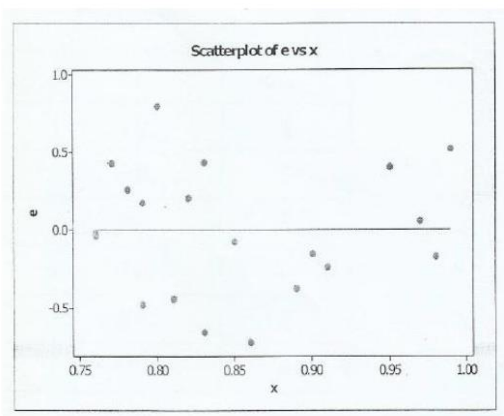
Example

The following data represent a store's share of a certain toothpaste (y) and its selling price (X) per pound for twenty consecutive months.

Test whether the errors are independent or autocorrelated.

الشهر	X	y	الشهر	X	y
1	0.97	3.36	11	0.79	7.25
2	0.95	4.2	12	0.83	6.09
3	0.99	3.33	13	0.81	6.8
4	0.91	4.54	14	0.77	8.65
5	0.98	2.89	15	0.76	8.43
6	0.90	4.87	16	0.80	8.29
7	0.89	4.9	17	0.83	7.18
8	0.86	5.29	18	0.79	7.9
9	0.85	6.18	19	0.76	8.45
10	0.82	7.2	20	0.78	8.23

1-We use the graphing method for the values of e_i versus the values of X_i as follows:



The graph shows that there is a positive autocorrelation between the errors, as there is a group of positive errors followed by negative errors, followed by positive errors.