Ex)

$$P(X) = \begin{cases} \frac{1}{6} & X = 1,2,3,4,5,6 \\ 0 & o.w. \end{cases}$$

Find the following:

$$E(X),\quad E(X^2),\quad E(X^4),\quad E\left(\frac{1}{X}\right),\quad E(X+4),\quad E(2X-1)^2$$

Sol)

$$E(X) = \sum_{x=1}^{6} X_i P(X_i) = \left[1 * \frac{1}{6} + 2 * \frac{1}{6} + 3 * \frac{1}{6} + \cdots + 6 * \frac{1}{6}\right]$$

$$= \frac{1}{6}[1+2+3+\cdots+6] = \frac{1}{6} * \frac{6(6+1)}{2} = \frac{7}{2}$$

Or $$= \frac{1}{6} * 21 = \frac{7}{2}$$

$$g(x) = X^2$$

$$E(X^2) = \sum_{x=1}^{6} g(x)P(x) = \sum_{x=1}^{6} X^2 \frac{1}{6} = \frac{1}{6}\sum_{x=1}^{6} X^2$$

$$= \frac{1}{6} * \frac{6(6+1)(2*6+1)}{6} = \frac{91}{6} = 15.1667$$

$$E(X^4) = \sum_{x=1}^{6} X^4 P(x) = \sum_{x=1}^{6} X^4 \frac{1}{6} = \frac{1}{6}\left[\frac{6(6+1)(2*6+1)(3*6+3*6-1)}{30}\right]$$

$$E\left(\frac{1}{X}\right) = \sum_{x=1}^{6} \frac{1}{X} P(x) = \sum_{x=1}^{6} \frac{1}{X} * \frac{1}{6} = \frac{1}{6}\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{6}\right]$$

And

$$E\left(\frac{1}{X}\right) \neq \frac{1}{E(x)}$$

$$E(X+4) = E(X) + 4 = \frac{7}{2} + 4 = \frac{15}{2}$$

$$E(2X-1)^2 = \sum_{x=1}^{6} (2X-1)^2 P(x) =$$

Or

$$E(4X^2 - 4X + 1) = 4E(X^2) - 4E(X) + 1$$

$$= 4 * \left(\frac{91}{6}\right) - 4 * \left(\frac{7}{2}\right) + 1$$

Ex)

$$f(X) = \begin{cases} \frac{1}{2} & 2 < X < 4 \\ 0 & o.w. \end{cases}$$

Find :

$$E(X), \quad E(X^2), \quad E\left(\frac{1}{X}\right), \quad E(X+1), \quad E(2X-1)^2$$

Sol)

$$E(X) = \int_{-\infty}^{\infty} X\, f(x) dx = \int_{2}^{4} X\, \frac{1}{2}\, dx = \frac{1}{2}(X^2|_2^4 = \frac{1}{2}(6) = 3$$

$$E(X^2) = \int_{-\infty}^{\infty} X^2\, f(x) dx = \int_{2}^{4} X^2\, \frac{1}{2}\, dx = \frac{1}{2}(\frac{X^3}{3}|_2^4 = \frac{56}{6}$$

$$E\left(\frac{1}{X}\right) = \int_{-\infty}^{\infty} \frac{1}{X}\, f(x) dx = \int_{2}^{4} \frac{1}{X}\, \frac{1}{2}\, dx = \frac{1}{2}(Ln\, X|_2^4 = ** \neq \frac{1}{E(X)}$$

$$E(X+1) = E(X) + 1 = 3 + 1 = 4$$

$$E(2X-1)^2 = E(4X^2 - 4X + 1) = 4E(X^2) - 4E(X) + 1$$

$$= 4 * \left(\frac{56}{6}\right) - 4 * (3) + 1 = 63.67$$

Ex) A dice is thrown once, where *X* will represent the money that a person wins or loses, as follows:

عدد ارقام الزار	$P(x)$	عدد النقود التي يربحها او يخسرها
1	1/6	−1
2	1/6	+2
3	1/6	+3
4	1/6	−4
5	1/6	+3
6	1/6	−6

قيمة سالبة (خسارة) $= E(x)$

قيمة موجبة (ربح) $= E(x)$

صفر (عدم وجود ربح او خسارة) $= E(x)$

Sol)

$$E(X) = \sum_{x=1}^{6} XP(X) = \left[\frac{-1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{-4}{6} + \frac{3}{6} + \frac{-6}{6}\right] = -\frac{1}{2}$$ خسارة

Ex) A box contains 4 red, 6 white and 8 green balls. If a red ball is drawn, he will win 3 dinars, but if a white ball is drawn, he will win 2 dinars, note that the person expected that he would get nothing, i.e., $E(X) = 0$, what is the value of k If he is drawn a green ball?

صندوق يحتوي على 4 كرات حمراء ، 6 كرات بيضاء ، 8 كرات خضراء ، فإذا تم سحب كرة حمراء سوف يربح 3 دنانير اما إذا سحب كرة بيضاء سوف يربح دينارين ، علما ان الشخص توقع عدم حصوله على أي شيء أي ان $E(X) = 0$ ، فما هي قيمة k اذا سحب كرة خضراء ؟

	عدد الدنانير	$P(x)$
حمراء	3	4/18
بيضاء	2	6/18
خضراء	K	8/18

$$\therefore E(x) = \sum x\,p(x)$$

$$0 = 3 \times \frac{4}{18} + 2 \times \frac{6}{18} + K \times \frac{8}{18}$$

$$\therefore K = -3$$

Ex) If we have :

$$f(X) = \begin{cases} 6X(1-X) & 0 < X < 1 \\ 0 & o.w. \end{cases}$$

Find : $E(X)$, $E\left(\frac{1}{1-X}\right)$

Sol)

$$E(X) = \int_{-\infty}^{\infty} X\, f(x)dx = \int_{0}^{1} X\, 6X(1-X)\, dx = \int_{0}^{1} 6(X^2 - X^3)\, dx =$$

$$= 6(\frac{X^3}{3} - \frac{X^4}{4}|_0^1 = 6\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{2}$$

$$E\left(\frac{1}{1-X}\right) = \int_{-\infty}^{\infty} \frac{1}{1-X}\, f(x)dx = \int_{0}^{1} \frac{1}{1-X}\, 6X(1-X)\, dx = \int_{0}^{1} 6X\, dx =$$

$$= 6(\frac{X^2}{2}|_0^1 = 6\left(\frac{1}{2}\right) = \ 3$$

4.2) **Variance:** التباين

Definition :

The Variance of a r.v. X is denoted by $V(x)\ or\ \sigma^2(x)$ and is defined by:

$$V(x) = \sigma^2(x) = E[X - E(X)]^2 = E(X^2) - [E(X)]^2$$

$$\therefore E(X) = \mu_x = mean$$

$$or\ E(Y) = \mu_y$$

$\sigma(x) = \sqrt{\sigma^2(x)}$ standard deviation

Proof:

$$V(x) = E(X - E(X))^2$$

$$\because E(X) = \mu_x$$

$$\therefore V(x) = E(X - \mu_x)^2 = E(X^2 - 2\mu_x X + {\mu_x}^2)$$

$$V(x) = E(X^2) - 2\mu_x E(X) + {\mu_x}^2$$

$$V(x) = E(X^2) - 2{\mu_x}^2 + {\mu_x}^2 = E(X^2) - {\mu_x}^2$$

$$= E(X^2) - (E(X))^2$$

Where