$$= \sum_{k=n}^{\infty} \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

$$= 1 - \sum_{k=0}^{n-1} \frac{(\lambda t)^k e^{-\lambda t}}{k!} \quad , n = 1, 2, \ldots \quad t \geq 0$$

The Probability density function of S_n is :

$$f_{S_n}(t) = \frac{d}{dt} F_{S_n}(t)$$

$$= \frac{d}{dt}\left[1 - e^{-\lambda t}\left(1 + \frac{\lambda t}{1!} + \frac{(\lambda t)^2}{2!} + \cdots + \frac{(\lambda t)^{n-1}}{(n-1)!}\right)\right]$$

$$= -e^{-\lambda t}\left[\lambda + \frac{\lambda(\lambda t)}{1!} + \frac{\lambda(\lambda t)^2}{2!} + \cdots + \frac{\lambda(\lambda t)^{n-2}}{(n-2)!}\right] + \lambda e^{-\lambda t}$$

$$\left[1 + \frac{\lambda t}{1!} + \frac{(\lambda t)^2}{2!} + \cdots + \frac{(\lambda t)^{n-1}}{(n-1)!}\right]$$

$$= -\lambda e^{-\lambda t}\left[1 + \frac{\lambda t}{1!} + \frac{(\lambda t)^2}{2!} + \cdots + \frac{(\lambda t)^{n-2}}{(n-2)!}\right] + \lambda e^{-\lambda t}\left[1 + \frac{\lambda t}{1!}\right.$$

$$\left. + \frac{(\lambda t)^2}{2!} + \cdots + \frac{(\lambda t)^{n-2}}{(n-2)!} + \frac{(\lambda t)^{n-1}}{(n-1)!}\right]$$

$$= \lambda e^{-\lambda t} \frac{(\lambda t)^{n-1}}{(n-1)!}$$

$$f_{S_n}(t) = \frac{\lambda^n t^{n-1} e^{-\lambda t}}{(n-1)!} \quad , n = 1, 2, \ldots \quad t \geq 0$$