3. Sub dividing a Poisson Process تقسيم أو تجزئة العملية البواسونية

Suppose that we look at how to break $\{X(t), t \geq 0\}$ a Poisson Process of rate λ into two Process $\{X_1(t), t \geq 0\}$ and $\{X_2(t), t \geq 0\}$ suppose that arrival in Process $\{X(t)\}$ is sent to the first Process with Prob (P) and to the second Process with Prob $(1-P)$, then the resulting Processes are each Poisson Process with rates $\lambda_1 = \lambda P$ and $\lambda_2 = \lambda(1-P)$ respectively with two Process are independent.

4. Poisson Process with binomial distribution

متى يتوزع بواسون إلى بايونميل

If $X(t)$ be a Poisson Process and let $s<t$, then:

$$P_r\{X(s)=k \mid X(t)=n\} = \frac{P_r\{X(s)=k \text{ and } X(t)=n\}}{P_r\{X(t)=n\}}$$

$$= \frac{P_r\{X(s)=k\} \text{ and } P_r\{X(t-s)=n-k\}}{P_r\{X(t)=n\}}$$

$$= \frac{\dfrac{e^{-\lambda s}(\lambda s)^k}{k!} \cdot \dfrac{e^{-\lambda(t-s)}[\lambda(t-s)]^{n-k}}{(n-k)!}}{\dfrac{e^{-\lambda t}(\lambda t)^n}{n!}}$$

$$= \frac{n!}{k!\,(n-k)!} \left(\frac{s}{t}\right)^{k} \left(1-\frac{s}{t}\right)^{n-k}$$

is a binomial dist with $P = \frac{s}{t}$.