Properties of Poisson Process خواص العملية البواسونية

1. Combining Poisson Process تركيب العملية البواسونية

Suppose that $\{X_1(t), t \geq 0\}$ and $\{X_2(t), t \geq 0\}$ are independent Poisson Process with rates λ_1, λ_2 respectively. Let $X(t) = X_1(t) + X_2(t)$ for all $t > 0$ then the Process $\{X(t), t \geq 0\}$ is consisting of all arrivals to both Process $X_1(t)$ and $X_2(t)$, then the Process $X(t)$ is a Poisson Process with rate $\lambda = \lambda_1 + \lambda_2$.

Proof: Since the Prob. generating function of $X_i(t)$ of Poisson dist is:-

$$P_{X_i(t)}(s) = e^{\lambda_i (s-1)t} \quad , \quad i = 1,2$$

Then the P.g.f of $X(t)$ is:

$$P_{X(t)}(s) = \prod_{i=1}^{2} P_{X_i(t)}(s)$$

$$= P_{X_1(t)}(s) \cdot P_{X_2(t)}(s)$$

$$= e^{\lambda_1 (s-1)t} \cdot e^{\lambda_2 (s-1)t}$$

$$= e^{(\lambda_1+\lambda_2)(s-1)t}$$

$$= e^{\lambda (s-1)t} \quad , \quad \lambda = \lambda_1 + \lambda_2$$

Thus $X(t)$ is a Poisson dist with rate $\lambda = \lambda_1 + \lambda_2$.

2. Difference of two independent Poisson Process.

الفرق بين عمليتين بواسون

Let $\{X_1(t)\}, \{X_2(t)\}$ be two indep. Poisson Process with rate λ_1, λ_2 respectively. Let $X(t) = X_1(t) - X_2(t)$ then the Process $\{X(t)\}$ has a dist given by:

$$P_r\{X(t)=n\} = e^{-(\lambda_1+\lambda_2)t} \left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{n}{2}} I_n\left(2t\sqrt{\lambda_1\lambda_2}\right)$$

When $n = 0, \mp 1, \mp 2, \ldots$ and:

$$I_n(x) = \sum_{r=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{2r+n}}{r! \, \Gamma(r+n-1)}$$

is the modified Bessel function of order $n \geq -1$.

1. Thus the difference of two independent Poisson Process is not Poisson Process.

2. The first moments of $X(t)$ are given by:

$$E[X(t)] = (\lambda_1 - \lambda_2)t$$

$$E[X^2(t)] = (\lambda_1+\lambda_2)t + (\lambda_1 - \lambda_2)^2 t^2$$