Solution: the committee contains is one of the same gender

① $C_4^{10} + C_4^6 = \frac{10!}{4!(10-4)!} + \frac{6!}{4!(6-4)!}$

② $C_1^{10} C_3^6 + C_2^{10} C_2^6 + C_3^{10} C_1^6 + C_4^{10} C_0^6$

$= 185$ ways

③ $C_2^{10} C_2^6 + C_1^{10} C_3^6 + C_0^{10} C_4^6$

$= 890$ ways

④ Excluding one of the women, then the number of women becomes 5

$C_3^5 C_1^{10} = \frac{5!}{3!(5-3)!} \frac{10!}{1!(10-1)!}$

⑤ Excluding one man, then then number of men becomes 9

$C_2^9 C_2^6 = \frac{9!}{2!(9-2)!} \frac{6!}{2!(6-2)!}$

$= 540$ way

Find

P_3^6, P_8^{10}, P_7^{15},

Solution

$$P_3^6 = \frac{6!}{(6-3)!} = \frac{6\times5\times4\times3!}{3!} = 120$$

$$P_8^{10} = \frac{10!}{(10-8)!} = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{2!}$$

$$= 1814400$$

$$P_7^{15} = \frac{15!}{(15-7)!}$$

If $P_2^n = 72$

Find n

$$P_r^n = \frac{n!}{(n-r)!}$$

$$72 = \frac{n!}{(n-2)!}$$

$$72 = \frac{n(n-1)(n-2)!}{(n-2)!}$$

$$72 = n^2 - n$$

$$n^2 - n - 72 = 0$$

$$(n-9)(n+8) = 0$$

$n = 9$

$n = -8$ neglated.

Find ①

$$\frac{1}{210}\left[P_3^7 + P_4^7\right] = \frac{1}{210}\left[\frac{7!}{(7-3)!} + \frac{7!}{(7-4)!}\right]$$

$$= \frac{1}{210}\left[\frac{7\times6\times5\times4!}{4!} + \frac{7\times6\times5\times4\times3!}{3!}\right]$$

$$= 5$$

② Find n if $C_{20}^{n} = C_{35}^{n}$

Since $C_r^n = C_{n-r}^n$

$\therefore r = 20$

$n - r = 35$

$n - 20 = 35$

$\therefore n = 20 + 35$

$= 55$

Example: If the number of questions of an exam are 10. we must choosen 4 from the first five

solution

$$C_4^5 C_3^5 = \frac{5!}{4!(5-4)!} \quad \frac{5!}{3!(5-3)!}$$

$= 50$ way

Example: Find the value of n if

$$2\binom{n}{2} = \binom{n+1}{3}$$

Solution

$$2C_2^n = \frac{2\,n!}{2!(n-2)!}$$

$$C_3^{n+1} = \frac{(n+1)!}{3!((n+1)-3)!}$$

$$= \frac{(n+1)!}{3!\,(n-2)!}$$

$$\frac{n!}{(n-2)!} = \frac{(n+1)!}{6\,(n-2)!}$$

$$n! = \frac{(n+1)(n+1-1)!}{6}$$

$$6\,n! = n!(n+1)$$

$$6 = n+1$$

$$n = 5$$

Example: If a student has to answer 8 questions out of 10, Find

① The number of ways a student can answer the equations

② the number of ways a student can answer the question if the first and second are compulsory

③ The number of ways than can be answered in order

Solution

① C_8^{10}

② $C_6^8 = \frac{8!}{6!(8-6)!} = 7 \times 4 = 28$

③ $3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

Example: Find the value of n if

$$\frac{n!}{(n-2)!} = 6$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 6$$

$$n^2 - n = 6$$

$$n^2 - n - 6 = 0$$

$$(n-3)(n+2) = 0 \Rightarrow n = 3$$