4.2 Poisson Process

The counting process $\{X(t), t \geq 0\}$ is said to be a Poisson process having rate (intensity) $\lambda > 0$ if :

1. $X(0) = 0$
2. For any time points $t_0=0<t1<t2<\ldots<t_n$,the process increments: $X(t_1)-X(t_0)$, $X(t_2)-X(t_1),\ldots$, $X(t_n)-X(t_{n-1})$ are independent increments.
3. The number of events in any interval X(t+s)-X(s) has a Poisson dist with mean λt , i.e :
   $P_r\{X(t+s)-X(s)=n\}=P_r\{x(t)=n\}$
   $$=\frac{(\lambda t)^n e^{-\lambda t}}{n!} \qquad n = 0,1,2,\ldots$$

Then since a Poisson process has stationary increments when:

$$E\{X(t)\}=\lambda t \quad , \quad var\{X(t)\}=\lambda t$$

Thus the expected number of events in an interval of unit length is λ.

4.3 Assumption of Poisson Process

In an interval of infinite length h, then:

1. The probability of exactly one event occurs in any short interval h is:
   $$P_r\{X_{t+h}-X_t=1\}=\lambda h + o(h) = P_1(h)$$

   Where $\lim_{h\to 0}\frac{o(h)}{h}=0$