Axiom I: P[A] $\geq 0 \quad \forall A \subset B$;

Axiom II: P[S] = 1

Axiom III: If $A_1, A_2, \ldots$ is an infinite sequence of events that are all incompatible when taken two at a time, then

$$P\left[\bigcup_{k=1}^{\infty} A_k\right] = \sum_{k=1}^{\infty} P[A_k]$$

In particular,

$$P(A \cup B) = P(A) + P(B) \qquad if\ A \cap B = \emptyset$$

Note:

1. $P(A^c)=1-P(A) \quad \forall\ A \subset S$
2. For all events A and B

$$P[A\cup B] = P[A] + P[B] - P[AB]$$

3. $P[A|B]=\frac{P[A\cap B]}{P[B]}$ if $P[B]>0$
4. $P[A \cap B] = P[A|B] \times P[B] = P[B|A] \times P[A] \qquad if\ P[A]P[B] > 0$

Definition(5):Conditional Probability

If A and B are any two events defined on the same sample space S, the conditional probability of A given B, is defined by:

$$P[A|B]=\frac{P[A\cap B]}{P[B]} \qquad \text{if} \quad P[B]>0$$

and if P[B]=0 then P[A|B] is undefined.

Definition(6):Independent events

Two events with nonzero probabilities are independent if and only if, any one of the following equivalent statements is true.

P[A|B]=P(A) P[B|A]=P(B) $P[A \cap B]$ ==P(A).P(B)

Definition(7):Random Variables

A random variable (r.v.) is a function X that associates a real number X(s) = x with each element s of S, where S is a sample space associated to a random experiment E. We denote by S_x the set of all possible values of X.

Definition(8): Distribution function

The distribution function of the r.v. X is defined by:

$$F_x(x) = P[X \le x] \quad \forall \quad x \in \mathbb{R}$$

Properties:

- $0 \le F_x(x) \le 1$
- $\lim_{x \to -\infty} F_x(x) = 0$
- $\lim_{x \to \infty} F_x(x) = 1$
- If $x_1 < x_2$ then $F_x(x_1) \le F_x(x_2)$
- P(a$< x < b$)= $F_x(b) - F_x(a)$