$$P(1 < y \leq 2) = F(2) - F(1) = 1 - \frac{10}{15} = \frac{5}{15}$$

$$P(y = 2)$$

Ex) Three balls are chosen at random out of (5) numbered ball from 2 to 6 , Let X be the sum of the three balls . Construct P.m.f. of X and F(x) .

Sol)

$2 \ , \ 3 \ , \ 4 \ , \ 5 \ , \ 6 \quad \therefore \ n = 5 \ , \ r = 3$

$$C_r^n = \frac{n!}{r!\,(n-r)!} = \frac{5!}{3!\,2!} = 10 \ \ way$$

$$S = \{234 \, , 235 \, , 236 \, , 245 \, , 246 \, , 256 \, , 345 \, , 346 \, , 356 \, , 456 \}$$

Sum = 9 10 11 11 12 13 12 13 14 15

x	9	10	11	12	13	14	15
$P(X = x)$	1/10	1/10	2/10	2/10	2/10	1/10	1/10

$$\sum_{i=9}^{15} P(x_i) = \frac{1}{10} + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{2}{10} + \frac{1}{10} + \frac{1}{10} = \frac{10}{10} = 1$$

$F(x) = P(X \leq x)$

$$F(x) = \begin{cases} 0 & x < 9 \\ 1/10 & 9 \leq x < 10 \\ 2/10 & 10 \leq x < 11 \\ 4/10 & 11 \leq x < 12 \\ 6/10 & 12 \leq x < 13 \\ 8/10 & 13 \leq x < 14 \\ 9/10 & 14 \leq x < 15 \\ 1 & x \geq 15 \end{cases}$$

Ex) Let X be a random variable (r.v.) with the following distribution . Find the cumulative distribution function(C.D.F.) of X and draw the diagram .

$$P(X) = \begin{cases} \frac{1}{4} & x = -2 \\ \frac{1}{8} & x = 1 \\ \frac{1}{2} & x = 2 \\ \frac{1}{8} & x = 4 \\ 0 & other\ wise \end{cases}$$

Sol)

$$F(x) = P(X \leq x)$$

$$\therefore F(x = -3) = P(X \leq -3) = P(X = -3) = 0$$

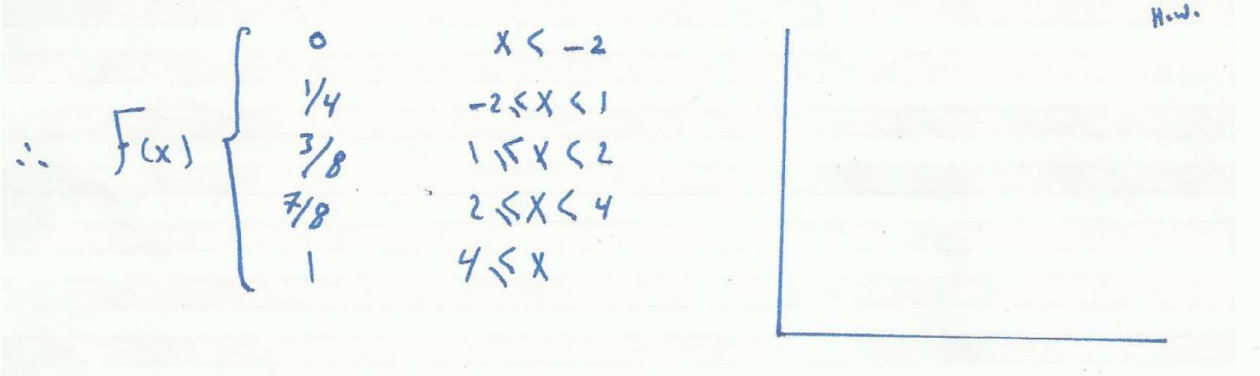
$$F(x = -2) = P(X \leq -2) = P(X = -2) = \frac{1}{4}$$

$$F(x = 1) = P(X \leq 1) = P(X = -2) + P(X = 1) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

$$F(x = 2) = P(X \leq 2) = P(X = -2) + P(X = 1) + P(X = 2) = \frac{3}{8} + \frac{1}{2} = \frac{7}{8}$$

$$F(x = 4) = P(X \leq 4) = P(X = -2) + P(X = 1) + P(X = 2) + P(X = 4) = \frac{7}{8} + \frac{1}{8} = \frac{8}{8} = 1$$

$$F(x = 5) = P(X \leq 5) = F(x = 4) + P(X = 5) = 1 + 0 = 1$$



Ex) A bowl contains three red chips and five blue chips . two chips are drawn at random from the bowl . Let X denote the number of red chips in sample . Find :-

1_ The (p.m.f.) of X and draw corresponding graph .

2_ The (c.p.f.) or (c.d.f.)of X and draw it

Sol)

1- the number of red chips in the sample of size

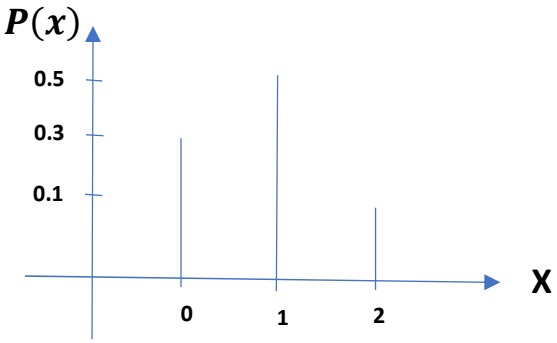
$X = 0\,,1\,,2$

$S = C_2^8 = \frac{8!}{2!\ 6!} = 28\ \ ways$

$P(X=0) = \frac{C_0^3\ C_2^5}{C_2^8} = \frac{10}{28}\ \ ,\ \ P(X=1) = \frac{C_1^3\ C_1^5}{C_2^8} = \frac{15}{28}$

$P(X=2) = \frac{C_2^3\ C_0^5}{C_2^8} = \frac{3}{28}$

x	0	1	2
$P(X=x)$	10/28	15/28	3/28



2- The (c.p.f.) or (c.d.f.)of X

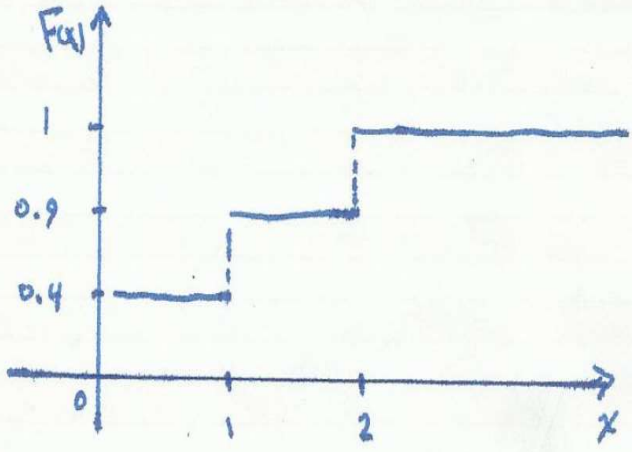
$F(x) = P(X \le x)$

$\therefore F(0) = P(X \le 0) = P(X=0) = \frac{10}{28}$

$F(1) = P(X \le 1) = P(X=0) + P(X=1) = \frac{10}{28} + \frac{15}{28} = \frac{25}{28}$

$F(2) = P(X \le 2) = P(X=0) + P(X=1) + P(X=2) = \frac{25}{28} + \frac{3}{28} = \frac{28}{28} = 1$

$$\therefore F(x) = \begin{cases} 0 & x < 0 \\ 10/28 & 0 \le x < 1 \\ 25/28 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$$



Ex) A r.v. X has the following p.m.f.

x	-2	-1	0	1	2	3
$P(X=x)$	K	2K	3K	5K	7K	2K

Find :

1- The value of K .

2- $P(X<1)$, $P(X\geq 0)$, $P(-1\leq X<2)$

Sol)

1- $\sum_{i=-2}^{3} P(x_i) = 1$

$\rightarrow K+2K+3K+5K+7K+2K=1$

$\rightarrow 20\,K=1 \quad \rightarrow \quad K=\frac{1}{20}$

2- a- $P(X<1) = P(X=0)+P(X=-1)+P(X=-2)$

$$=\frac{3}{20}+\frac{2}{20}+\frac{1}{20}=\frac{6}{20}$$

b- $P(X\geq 0) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$$=\frac{3}{20}+\frac{5}{20}+\frac{7}{20}+\frac{2}{20}=\frac{17}{20}$$

c- $P(-1\leq X<2) = P(X=-1)+P(X=0)+P(X=1)$

$$=\frac{2}{20}+\frac{3}{20}+\frac{5}{20}=\frac{10}{20}$$

Note :-

$\{P(X\leq a)=1-P(X>a) \;;\; P(X>a)=P(X\leq a)=1\}$

$\{P(X\geq 0)=1-P(X<0) \}$