Theorem(1): under the assumption of Poisson Process.

then $X(t)$ follow the Poisson distribution:

$$P_n(t) = P_r\{X(t)=n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \qquad n=0,1,\ldots$$

Proof: Consider $P_n(t+h)$ for $h \geq 0$, n events by instant Can happen in the following way:

$$P_n(t+h) = P_r\{X(t+h)=n\}$$

$$\begin{aligned} P_n(t+h) = P_r\{ & X(t)=n \text{ and } X(h)=0 \text{ or} \\ & X(t)=n-1 \text{ and } X(h)=1 \text{ or} \\ & X(t)=n-k \text{ and } X(h)=k, \quad k=2,3,\ldots\} \end{aligned}$$

$$\begin{aligned} P_n(t+h) = {} & P_r\{X(t)=n\} \cdot P_r\{X(h)=0 \mid X(t)=n\} + P_r\{X(t)=n-1\} \cdot \\ & P_r\{X(h)=1 \mid X(t)=n-1\} + P_r\{X(t)=n-k\} \cdot P_r\{X(h)=k \mid X(t)=n-k\} \end{aligned}$$

$$P_n(t+h) = P_n(t) \cdot P_0(h) + P_{n-1}(t) \cdot P_1(h) + o(h)$$