y^2	X_2y	X_1y	X_2^2	X_1^2	X_1X_2	X_2	X_1	y	المشاهدة
36	6	18	1	9	3	1	3	6	1
25	10	25	4	25	10	2	5	5	2
64	40	32	25	16	20	5	4	8	3
49	49	63	49	81	63	7	9	7	4
36	54	12	81	4	18	9	2	6	5
4	16	2	64	1	8	8	1	2	6
214	175	152	224	136	122	32	24	34	المجموع

We substitute these sums into the matrix $(X'X)$ and vector $X'y$ as follows:

$$(X'X) = \begin{bmatrix} 6 & 24 & 32 \\ 24 & 136 & 122 \\ 32 & 122 & 224 \end{bmatrix}, X'y = \begin{bmatrix} 34 \\ 152 \\ 175 \end{bmatrix}$$

Thus, the vector of estimated parameters is as follows:

$$\hat{\beta} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} = (X'X)^{-1}X'y = \begin{bmatrix} 6 & 24 & 32 \\ 24 & 136 & 122 \\ 32 & 122 & 224 \end{bmatrix}^{-1} \begin{bmatrix} 34 \\ 152 \\ 175 \end{bmatrix}$$

The inverse of the matrix $(X'X)$ is found and then the resulting matrix is multiplied by the vector $X'y$ as follows:

$$(X'X)^{-1} = \begin{bmatrix} 1.238 & -0.1169 & -0.1131 \\ -0.1169 & 0.0254 & 0.00286 \\ -0.1131 & 0.00286 & 0.01907 \end{bmatrix}$$

Thus, the vector of parameters is as follows:

$$\hat{\beta} = (X'X)^{-1}X'y = \begin{bmatrix} 1.238 & -0.1169 & -0.1131 \\ -0.1169 & 0.0254 & 0.00286 \\ -0.1131 & 0.00286 & 0.01907 \end{bmatrix} \begin{bmatrix} 34 \\ 152 \\ 175 \end{bmatrix}$$

$$= \begin{bmatrix} 4.5117 \\ 0.3889 \\ -0.075 \end{bmatrix}$$

Thus, the estimated regression line equation is as follows:

$$\hat{y}_i = 4.5117 + 0.3889X_1 - 0.075X_2$$

2- Find the variance of the estimated parameters $V(\hat{\beta})$

The formula for the variance of the estimated parameters is:

$$V(\hat{\beta}) = \hat{\sigma}^2 (X'X)^{-1}$$

The estimated population variance is:

$$\hat{\sigma}^2 = MSe = \frac{y'y - \hat{\beta}'X'y}{n-m-1}$$

$$y'y = \sum y_i^2 = 214$$

$$\hat{\beta}'X'y = [\hat{\beta}_0 \quad \hat{\beta}_1 \quad \hat{\beta}_2] \begin{bmatrix} \sum y_i \\ \sum X_i y_i \\ \sum X_i y_i \end{bmatrix}$$

$$\hat{\beta}'X'y = [4.5117 \quad 0.887 \quad -0.075] \begin{bmatrix} 34 \\ 152 \\ 175 \end{bmatrix}$$

$$\hat{\beta}'X'y = 199.3617$$

So:

$$\hat{\sigma}^2 = MSe = \frac{y'y - \hat{\beta}'X'y}{n-m-1} = \frac{214 - 199.3617}{6-2-1} = 4.879$$

Estimated population variance:

$$V(\hat{\beta}) = \hat{\sigma}^2 (X'X)^{-1}$$

$$= 4.879 \begin{bmatrix} 1.238 & -0.1169 & -0.1131 \\ -0.1169 & 0.0254 & 0.00286 \\ -0.1131 & 0.00286 & 0.01907 \end{bmatrix}$$

$$= \begin{bmatrix} 6.0405 & -0.5707 & -0.5521 \\ -0.5707 & 0.12406 & 0.01395 \\ -0.5521 & 0.01395 & 0.09305 \end{bmatrix}$$

Thus:

$$S^2_{\hat{\beta}} = Mse \begin{bmatrix} C_{00} & C_{01} & C_{02} \\ C_{10} & C_{11} & C_{12} \\ C_{20} & C_{21} & C_{22} \end{bmatrix}$$

$$= 4.879 \begin{bmatrix} 1.238 & -0.1169 & -0.1131 \\ -0.1169 & 0.0254 & 0.00286 \\ -0.1131 & 0.00286 & 0.01907 \end{bmatrix}$$

$$S^2_{\hat{\beta}} = \begin{bmatrix} 6.0405 & -0.5707 & -0.5521 \\ -0.5707 & 0.12406 & 0.01395 \\ -0.5521 & 0.01395 & 0.09305 \end{bmatrix}$$

Thus, the variances of the estimated coefficients are as follows:

$$S^2_{\hat{\beta}_0} = 6.0405$$

$$S^2_{\hat{\beta}_1} = 0.12406$$

$$S^2_{\hat{\beta}_2} = 0.09305$$

The covariance between the estimated parameters is found as follows:

$$Cov(\hat{\beta}_0, \hat{\beta}_1) - 0.5707$$

$$Cov(\hat{\beta}_0, \hat{\beta}_2) - 0.5521$$

$$Cov(\hat{\beta}_1, \hat{\beta}_2) = 0.01395$$

The variances of the estimated coefficients can also be calculated using the matrix C and Mse as follows:

$$S^2_{\hat{\beta}_0} = Mse\,C_{00} \;,\; S^2_{\hat{\beta}_1} = Mse\,C_{11} \;,\; S^2_{\hat{\beta}_2} = Mse\,C_{22}$$

That is:

$$S^2_{\hat{\beta}_0} = Mse * C_{00} = (4.879) * (1.238) = 6.0405$$

$$S^2_{\hat{\beta}_1} = Mse * C_{11} = (4.879) * (0.0254) = 0.12406$$

$$S^2_{\hat{\beta}_2} = Mse * C_{22} = (4.879) * (0.01907) = 0.09305$$

The covariance between the estimated parameters can also be calculated using the matrix C and Mse as follows:

$$Cov(\hat{\beta}_0, \hat{\beta}_1) = Mse\, C_{01}\ ,\ Cov(\hat{\beta}_0, \hat{\beta}_2) = Mse\, C_{02}\ ,\ Cov(\hat{\beta}_1, \hat{\beta}_2) = Mse\, C_{12}$$

That is:

$$Cov(\hat{\beta}_0, \hat{\beta}_1) = Mse * C_{01} = (4.879) * (-0.11697) = -0.5707$$

$$Cov(\hat{\beta}_0, \hat{\beta}_2) = Mse * C_{02} = (4.879) * (-0.1131) = -0.5521$$

$$Cov(\hat{\beta}_1, \hat{\beta}_2) = Mse * C_{12} = (4.879) * (0.00286) = 0.01395$$