EX$_2$: if you have number defined $\mathbf{a = [10 \quad 100]}$ find the final $(\mathbf{W})$ to this vector.

a_{11}	a_{12}	y	W_1	W_2	$\hat{y}$	error	W_1new	W_2new	$\hat{y}$	error
10	100	1	0	0	0	1	1	1	1	0
100	10	1	1	1	1	0	1	1		
10	10	1	1	1	1	0	1	1		
100	100	1	1	1	1	0				

1) $\mathbf{y = \sum a_{ij} = a_{11} + a_{12}} \Longrightarrow (10 + 100) = 110 > 0 = \mathbf{1}$

$W_1 = 0\,, \quad W_2 = 0$

$\mathbf{\hat{y} = f \sum W_i a_i = f(W_1 a_{11} + W_2 a_{12})} \Longrightarrow f(0 * 10 + 0 * 100) = f(0) = \mathbf{0}$

$\mathbf{error = y - \hat{y}} \Longrightarrow 1 - 0 = \mathbf{1}$

$\mathbf{W_1 new = W_1 old + \alpha(y - \hat{y})a_{11}} \Longrightarrow 0 + 1(1)10 = 10 > 0 = \mathbf{1}$

$\mathbf{W_2 new = W_2 old + \alpha(y - \hat{y})a_{12}} \Longrightarrow 0 + 1(1)100 = 100 > 0 = \mathbf{1}$

$\mathbf{\hat{y} = f(W_1 a_{11} + W_2 a_{12})} \Longrightarrow f(1 * 10 + 1 * 100) = f(110) > 0 = \mathbf{1}$

$\mathbf{error = y - \hat{y}} \Longrightarrow 1 - 1 = \mathbf{0}$

2) $\mathbf{y = \sum a_{ij} = a_{11} + a_{12}} \Longrightarrow (100 + 10) = 110 > 0 = \mathbf{1}$

$\mathbf{\hat{y} = f \sum W_i a_i = f(W_1 a_{11} + W_2 a_{12})} \Longrightarrow f(1 * 100 + 1 * 10) = f(110) > 0 = \mathbf{1}$

$\mathbf{error = y - \hat{y}} \Longrightarrow 1 - 1 = \mathbf{0}$

$\mathbf{W_1 new = W_1 old + \alpha(y - \hat{y})a_{11}} \Longrightarrow 1 + 1(0)100 = 10 > 0 = \mathbf{1}$

$\mathbf{W_2 new = W_2 old + \alpha(y - \hat{y})a_{12}} \Longrightarrow 1 + 1(0)10 = 100 > 0 = \mathbf{1}$

$\mathbf{\hat{y} = f(W_1 a_{11} + W_2 a_{12})} \Longrightarrow f(1 * 10 + 1 * 100) = f(110) > 0 = \mathbf{1}$

3) $\mathbf{y = \sum a_{ij} = a_{11} + a_{12}} \Longrightarrow (10 + 10) = 20 > 0 = \mathbf{1}$

$\mathbf{\hat{y} = f \sum W_i a_i = f(W_1 a_{11} + W_2 a_{12})} \Longrightarrow f(1 * 10 + 1 * 10) = f(20) > 0 = \mathbf{1}$

$\mathbf{error = y - \hat{y}} \Longrightarrow 1 - 1 = \mathbf{0}$

$\mathbf{W_1 new = W_1 old + \alpha(y - \hat{y})a_{11}} \Longrightarrow 1 + 1(0)10 = 10 > 0 = \mathbf{1}$

$\mathbf{W_2 new} = \mathbf{W_2 old} + \boldsymbol{\alpha}(\mathbf{y} - \hat{\mathbf{y}})\mathbf{a_{12}} \Longrightarrow 1 + 1(0)10 = 100 > 0 = \mathbf{1}$

$\hat{\mathbf{y}} = \mathbf{f}(\mathbf{W_1 a_{11}} + \mathbf{W_2 a_{12}}) \Longrightarrow f(1 * 10 + 1 * 100) = f(110) > 0 = \mathbf{1}$

4) $\mathbf{y} = \sum \mathbf{a_{ij}} = \mathbf{a_{11}} + \mathbf{a_{12}} \Longrightarrow (100 + 100) = 200 > 0 = \mathbf{1}$

$\hat{\mathbf{y}} = \mathbf{f} \sum \mathbf{W_i a_i} = \mathbf{f}(\mathbf{W_1 a_{11}} + \mathbf{W_2 a_{12}}) \Longrightarrow f(1 * 100 + 1 * 100) = f(200) > 0 = \mathbf{1}$

$\mathbf{error} = y - \hat{y} = 1 - 1 = \mathbf{0}$

EX3: if $X = [0 \quad 1]$ solve this vector to find the output resalt.

X_{11}	X_{12}	y	W_1	W_2	$\hat{y}$	error	W_1new	W_2new	$\hat{y}$	error
0	1	1	0	0	0	1	0	1	1	0
1	0	1	0	1	0	1	1	1	1	0
0	0	0	1	1	0	0	1	1		
1	1	1	1	1	1	0				

1) $y = \sum X_{ij} = X_{11} + x_{12} = 0 + 1 = 1$

$W_1 = 0 , W_2 = 0$

$\hat{y} = f \sum W_i X_i = f(W_1 X_{11} + W_2 X_{12}) \Longrightarrow f(0 * 0 + 0 * 1) = f(0) = 0$

$error = y - \hat{y} \Longrightarrow 1 - 0 = 1$

$W_1 new = W_1 old + \alpha(y - \hat{y}) X_{11} \Longrightarrow 0 + 1(1)0 = 0$

$W_2 new = W_2 old + \alpha(y - \hat{y}) X_{12} \Longrightarrow 0 + 1(1)1 = 1$

$\hat{y} = f(W_1 X_{11} + W_2 X_{12}) \Longrightarrow f(0 * 0 + 1 * 1) = f(1) = 1$

$error = y - \hat{y} \Longrightarrow 1 - 1 = 0$

2) $y = \sum X_{ij} = X_{11} + X_{12} = 1 + 0 = 1$

$\hat{y} = f \sum W_i X_i = f(W_1 X_{11} + W_2 X_{12}) \Longrightarrow f(0 * 1 + 1 * 0) = f(0) = 0$

$error = y - \hat{y} = 1 - 0 = 1$

$W_1 new = W_1 old + \alpha(y - \hat{y}) X_{11} \Longrightarrow 0 + 1(1)1 = 1$

$W_2 new = W_2 old + \alpha(y - \hat{y}) X_{12} \Longrightarrow 1 + 1(1)0 = 1$

$\hat{y} = f(W_1 X_{11} + W_2 X_{12}) \Longrightarrow f(1 * 1 + 1 * 0) = f(1) = 1$

$error = y - \hat{y} \Longrightarrow 1 - 1 = 0$

3) $y = \sum x_{ij} = x_{11} + x_{12} \Longrightarrow 0 + 0 = 0$

$\hat{y} = f(W_1 X_{11} + W_2 X_{12}) \Longrightarrow f(1 * 0 + 1 * 0) = f(0) = 0$

$$\textbf{error} = y - \hat{y} = 0 - 0 = \mathbf{0}$$

$$\mathbf{W_1 new} = \mathbf{W_1 old} + \boldsymbol{\alpha}(\mathbf{y} - \hat{\mathbf{y}})\mathbf{X_{11}} \Longrightarrow 1 + 1(0)0 = \mathbf{0}$$

$$\mathbf{W_2 new} = \mathbf{W_2 old} + \boldsymbol{\alpha}(\mathbf{y} - \hat{\mathbf{y}})\mathbf{X_{12}} \Longrightarrow 1 + 1(0)0 = \mathbf{1}$$

$$\textbf{4)}\quad \mathbf{y} = \sum \mathbf{X_{ij}} = \mathbf{X_{11}} + \mathbf{X_{12}} \Longrightarrow 1 + 1 = 2 > 0 = \mathbf{1}$$

$$\hat{\mathbf{y}} = \mathbf{f}(\mathbf{W_1 X_{11}} + \mathbf{W_2 X_{12}}) \Longrightarrow f(1 * 1 + 1 * 1) = f(2) > 0 = \mathbf{1}$$

$$\textbf{error} = y - \hat{y} \Longrightarrow 1 - 1 = \mathbf{0}$$