Ex) let the (P. m. f.) of the r.v. X is given by :

$$\boldsymbol{P(x)} = \begin{cases} \frac{1}{3} & ; \quad \boldsymbol{x = 0, 1, 2} \\ \boldsymbol{0} & ; \quad other\ wise \end{cases}$$

1- Find (c.p.f.) or (c.d.f.) or (D.F.) or $F(X)$ and draw it :

2- Find $P(X > 0)$; $P(X \le 1); P(X \le 0)$; $P(|X| < 1);\ P(|X| + 1 < 2); P(X > 1)$

Sol) 1- $F(X) = P(X \le x)$

$F(0) = P(X < 0)$

$$F(0) = P(X \le 0) = P(X = 0) = \frac{1}{3}$$

$$F(1) = P(X \le 1) = P(X = 0) + P(X = 1) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$F(2) = P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$\therefore F(X) = \begin{cases} 0 & X < 0 \\ \frac{1}{3} & 0 \le X < 1 \\ \frac{2}{3} & 1 \le X < 2 \\ 1 & 2 \le X \end{cases}$$

2- i- $P(X > 0) = P(X = 1) + P(X = 2) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

ii- $P(X \le 1) = P(X = 1) + P(X = 0) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

iii- $P(X \le 0) = P(X = 0) = \frac{1}{3}$

iv- $P(|X| < 1) = P(-1 < X < 1) = P(0 \le X < 1) = P(X = 0) = \frac{1}{3}$

or $P(-1 < X < 0) + P(0 \le X < 1) = P(X = 0) = \frac{1}{3}$

v- $P(|X| + 1 < 2) = P(|X| < 1) = P(-1 < X < 1) = \frac{1}{3}$

vi- $P(X > 1) = P(X = 2) = \frac{1}{3}$

3- Probability Density Function (p.d.f.) دالة الكثافة الاحتمالية

Definition :

The probability density function of continuous r.v. X is a function (f) which the probability that the area under the curve of this function corresponding to any interval is equal to the probability that X will have a value in this interval . Then $f(X)$ is called the probability density function (p.d.f.) .

Properties :

1- $f(X) \geq 0$; $0 \leq f(X) \leq 1$

2- $\int_{-\infty}^{\infty} f(X)\, dx = 1$

3- For any interval [a,b]

$$P(a \leq x \leq b) = \int_a^b f(x)dx = F(b) - F(a)$$

$$F(X) = \int_{-\infty}^{x} f(t)dt \quad or \quad F(Y) = \int_{-\infty}^{y} f(t)dt$$

$$P(X \leq y) = P(X < y) = \int_{-\infty}^{y} f(x)dx$$

Notes :

1- $P(a \leq x \leq b) = P(a < x \leq b) = P(a \leq x < b) = P(a < x < b) = \int_a^b f(x)dx$

2- $P(X = a) = zero$ لأنه لا يوجد مساحة للمنحنى عند نقطة واحدة

3- $P(X \leq a) = P(X < a) = \int_{-\infty}^{a} f(x)dx$

4- $P(X > a) = P(X \geq a) = \int_a^{\infty} f(x)dx$

5- $P(X > a) = 1 - P(X \leq a)$

$P(X < a) = 1 - P(X \geq a)$

$P(X \leq a) = 1 - P(X > a)$

الملاحظة الخامسة تستخدم في حالة المتغير المستمر و/ او المتقطع

Ex) Let the r. v. X have the (p.d.f)

$$f(x) = \begin{cases} 2x & ; \quad 0 < x < 1 \\ 0 & ; \quad o.w. \end{cases}$$

Find : $P\left(\frac{1}{2} < x < \frac{3}{4}\right)$ & $P\left(-\frac{1}{2} < x < \frac{1}{2}\right)$

Sol)

1- $P\left(\frac{1}{2} < x < \frac{3}{4}\right) = \int_{\frac{1}{2}}^{\frac{3}{4}} f(x)dx = \int_{\frac{1}{2}}^{\frac{3}{4}} 2x = 2\frac{x^2}{2}\Big|_{\frac{1}{2}}^{\frac{3}{4}} = \left(\frac{3}{4}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{9}{13} - \frac{1}{4} = \frac{5}{16}$

Or $P(a < x < b) = F(b) - F(a)$

2- $P\left(-\frac{1}{2} < x < \frac{1}{2}\right) = P\left(-\frac{1}{2} < x < 0\right) + P\left(0 < x < \frac{1}{2}\right)$

تم التجزئة لكون الفترة من $0 < x < 1$ وان $-\frac{1}{2}$ تقع خارج هذه الفترة لذلك نلجئ الى التجزئة

$$\int_{-\frac{1}{2}}^{0} f(x)dx + \int_{0}^{\frac{1}{2}} f(x)dx \rightarrow \int_{-\frac{1}{2}}^{0} 2x\, dx + \int_{0}^{\frac{1}{2}} 2x\, dx$$

$$0 + 2\frac{x^2}{2}\Big|_0^{\frac{1}{2}} = \left(\frac{1}{2}\right)^2 - 0 = \frac{1}{4}$$

3- (p.d.f) هل ان الدالة هي

$\rightarrow \int_0^1 f(x)dx = \int_0^1 2x\, dx = 2\frac{x^2}{2}\Big|_0^1 = 1 - 0 = 1$ تحقق الشرط

4- $P(X \le 0.3) = P(X < 0.3) = \int_0^{0.3} 2x\, dx = 2\frac{x^2}{2}\Big|_0^{0.3} = (0.3)^2 = 0.09$

$\rightarrow P(X > 0.3) = \int_{0.3}^{1} 2x\, dx = 2\frac{x^2}{2}\Big|_{0.3}^{1} = 1 - (0.3)^2 = 1 - 0.09 = 0.91$

Or $1 - P(X \le 0.3)$

4- Distribution Function

Definition :

The distribution function of continuous random variable is denoted by $F(X)$ and is defined by :

$$F(X) = P(X \le x) = \int_{-\infty}^{x} f(t)dt$$

Properties

1_ $F(\infty) = \lim_{x\to\infty} F(x) = 1$

2_ $F(-\infty) = \lim_{x\to-\infty} F(x) = 0$

3_ $F(x_1) \le F(x_2) \quad ; \quad if \;\; x_1 \le x_2$

4_ $P(a \le x \le b) = F(b) - F(a)$

$$P(x > b) = 1 - F(b)$$

Ex) من المثال السابق

5- $F(X) = P(X \le x) = \int_{-\infty}^{x} f(t)dt = \int_0^x 2t\, dt = 2\frac{t^2}{2}|_0^x$

$\therefore F(X) = x^2$

$\therefore P\left(\frac{1}{2} < x < \frac{3}{4}\right) = F\left(\frac{3}{4}\right) - F\left(\frac{1}{2}\right) = \frac{9}{16} - \frac{1}{4} = \frac{5}{16}$

Ex) (H.W.) من المثال السابق جد ما يلي

1- $P\left(X < \frac{1}{2}\right) = \int_0^{\frac{1}{2}} 2x\, dx$

2- $P(X > 0.3) = \int_{0.3}^{1} 2x\, dx$

3- $P(|X| < 1) = P(-1 < X < 1) = \int_{-1}^{0} 2x\, dx + \int_0^1 2x\, dx$

4- $P(0.2 < X < 0.3) = \int_{0.2}^{0.3} 2x\, dx \;\; or \;\; F(0.3) - F(0.2)$

5- $P(-1 < X < 0.3)$

6- $P(0.4 < X < 3)$

7- $P(X = 0.5)$

$$\therefore F(X) = \begin{cases} 0 & X \le 0 \\ x^2 & 0 < X < 1 \\ 1 & X \ge 1 \end{cases}$$

ملاحظة : في حالة وجود في السؤال فقط $F(X)$ (دالة التوزيع) والمطلوب إيجاد دالة الكثافة الاحتمالية فيتم اشتقاق الدالة للحصول على (p.d.f.)