Chapter three: Markov Chain

A special kind of Markov processes is a Markov chain; so in this chapter we consider the class of Markov processes in discrete time (T) with a discrete state space(S). We call such processes **Markov chains**. Thus we may define a Markov chain as a sequence $X_0, X_1, \ldots$ of discrete random variables with the property that the conditional distribution of X_{n+1} given $X_0, X_1, \ldots, X_n$ depends only on the value of X_n but not further on $X_0, X_1, \ldots, X_{n-1}$.

i.e. for any set of values *h, j, ... , i* belonging to the discrete state space:

$$\Pr\{ X_{n+1} = j \mid X_0 = h, X_1 = k \ldots, X_n = i\}$$

$$= \Pr\{ X_{n+1} = j \mid X_n = i, \ldots, X_1 = k, X_0 = h \}$$

$$= \Pr\{ X_{n+1} = j \mid X_n = i\}$$

$$= \Pr\{ X_{n+1} = j \mid X_n = i\} = P_{ij} \qquad \ldots(1)$$

and this called a Markovian property.

The probability P_{ij} is the probability of transition from state(i) at n^{th} trail to state(j) at $n+1^{th}$ trail and must satisfy the following conditions:

1. $\sum_{j=0}^{\infty} P_{ij} = 1 \qquad i=1,2,\ldots$
2. $P_{ij} \geq 0$

3.2 Transition Probability Matrix

The one-step transition probability matrix **P** of a Markov chain is given by:

$$P=\begin{array}{c} \\ 0 \\ 1 \\ \vdots \\ n \end{array}\begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & \dots & n \end{array} \\ \begin{bmatrix} p_{00}p_{01}p_{02} & \cdots & p_{0n} \\ p_{10}p_{11}p_{12} & \cdots & p_{1n} \\ \vdots & \dots & \vdots \\ p_{n0}p_{n1}p_{n2} & \cdots & p_{nn} \end{bmatrix} \end{array}$$

This is the ***transition matrix*** and the probabilities P_{ij} should satisfy the following conditions:

1. $\sum_{j=0}^{\infty} P_{ij} = 1$ $\qquad$ i=1,2,…
2. $P_{ij} \geq 0$

The elements will all be non-negative and the rows all sum to unity: a matrix with the latter property is often called ***a stochastic matrix.***
Any stochastic matrix is a transition matrix of *M.C*.

Returning to the two previous examples, the state space of the *M.C* and transition matrix can be defined for each of them as follows:

Example(1): S={s,t}

$$P=\begin{array}{c} \\ t \\ c \end{array}\begin{array}{c} \begin{array}{cc} t & c \end{array} \\ \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix} \end{array}$$