$$= P_n(t)(1-\lambda h) + P_{n-1}(t)\lambda h + o(h) \quad , n \geq 1$$

$$P_n(t+h) - P_n(t) = -\lambda h P_n(t) + \lambda h P_{n-1}(t) + o(h)$$

بقسمة الطرفين على h

$$\frac{P_n(t+h) - P_n(t)}{h} = -\lambda P_n(t) + \lambda P_{n-1}(t) + \frac{o(h)}{h}$$

$$= -\lambda \left[P_n(t) - P_{n-1}(t) \right] + \frac{o(h)}{h}$$

In the limit as $h \to 0$, we have:

$$P_n'(t) = -\lambda \left[P_n(t) - P_{n-1}(t) \right] \quad , n \neq 1 \quad \text{since } \lim_{h\to 0} \frac{o(h)}{h} = 0$$

For $n = 0$ we have:

$$P_0(t+h) = P_r\{X(t+h) = 0\}$$

$$= P_r\{X(t) = 0 \text{ and } X(t+h) - X(t) = 0\}$$

$$= P_r\{X(t) = 0\} \cdot P_r\{X(t+h) - X(t) = 0\}$$

$$= P_0(t)(1 - \lambda h + o(h))$$