$$\hat{\sigma}^2 = S^2_{y/x} = MSe = V(y_i) = \frac{SSe}{n-2} = \frac{S_{yy} - \hat{\beta}_1 S_{Xy}}{n-2}$$

Or:

$$\hat{\sigma}^2 = S^2_{y/x} = MSe = V(y_i) = \frac{SSe}{n-2} = \frac{S_{yy} - \hat{\beta}_1^2 S_{XX}}{n-2}$$

We can use one of the two formulas.

Using the first formula:

We find first S_{yy}

$$S_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n}$$
$$= 202094 - \frac{(1410)^2}{10}$$
$$= 3284$$

Then we find S_{Xy}

$$S_{Xy} = \sum X_i y_i - \frac{(\sum y_i)(\sum X_i)}{n}$$
$$S_{Xy} = 71566 - \frac{(1410)(491)}{10}$$
$$S_{Xy} = 2335$$

Thus:

$$\hat{\sigma}^2 \frac{SSe}{n-2} = \frac{3284 - (1.1396)(2335)}{10-2}$$
$$\hat{\sigma}^2 = 77.87$$

Using the second formula:

We find S_{XX}:

$$S_{XX} = \sum X_i^2 - \frac{(\sum X_i)^2}{n}$$
$$= 26157 - \frac{(491)^2}{10}$$
$$= 2048.9$$

Thus:

$$\hat{\sigma}^2 = \frac{SSe}{n-2} = \frac{3284 - (1.1396)^2(2048.9)}{10-2}$$

$$\hat{\sigma}^2 = 77.87$$

5- We find the variance estimate for each of the following β_1, β_0.

We find the estimate of the variance of β_0 first:

$$V(\hat{\beta}_0) = \hat{\sigma}^2 \frac{\sum X_i^2}{nS_{XX}}$$

$$V(\hat{\beta}_0) = (77.9)\frac{(26157)}{(10)(2048.9)}$$

$$V(\hat{\beta}_0) = 99.4$$

Then we find the estimate of the variance of β_1

$$V(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{S_{XX}}$$

$$V(\hat{\beta}_1) = \frac{77.9}{2048.9}$$

$$V(\hat{\beta}_1) = 0.038$$

Test Hypothesis

Hypothesis testing is testing whether a sample returns to a specific population, i.e. whether it returns to the same population or to another population.
Or testing whether the sample returns to a population with a specific value.
There are two types of hypotheses:

1- Null Hypothesis:

It is symbolized by the symbol H_0 and is related to the parameter under study and it determines the values that are believed to express the true value of the parameter.

2- Alternative Hypothesis:

It is symbolized by the symbol H_1 and it determines the values of the parameter that are believed to be correct, and we hope that the sample data will lead to accepting the alternative hypothesis H_1 on the basis that it is a correct hypothesis.

A- Testing the hypotheses related to β_0

1- To test the hypothesis that β_0 is equal to zero is:

$$H_0 : \beta_0 = 0$$
$$H_1 : \beta_0 \neq 0$$

To test this hypothesis, we use the t-statistic, which takes the following form

$$t = \frac{\hat{\beta}_0}{S_{\hat{\beta}_0}}$$

The t-statistical test is distributed with a t-distribution with (n-2) degrees of freedom. If the calculated value is smaller than the tabular value, that is:

$$|t_{cal.}| < t(\frac{\alpha}{2}, n-2)$$

Then the null hypothesis H_0 is accepted and the alternative hypothesis is rejected. If the calculated value is greater than the table value, that is:

$$|t_{cal.}| > t(\frac{\alpha}{2}, n-2)$$

Then you reject the null hypothesis H_0 and accept the alternative hypothesis H_1.

Example:

Test β_0 equals zero for the previous example when $\alpha = 0.05$ (significance level).

Solution:

The hypothesis is written:

$$H_0 : \beta_0 = 0$$
$$H_1 : \beta_0 \neq 0$$

The degree of freedom to extract the tabular t-value from the respective tables of the t-distribution is calculated as follows:

$$n-2$$
$$10-2=8$$

The significance level

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

From the t-distribution tables, the tabular value is as follows:

$t(0.025, 8) = 2.306$

The t-statistic is now calculated:

$$t = \frac{\hat{\beta}_0}{S_{\hat{\beta}_0}} = \frac{85.043}{S_{\hat{\beta}_0}}$$

Where $S_{\hat{\beta}_0}$ is the square root of the variance of the estimated parameter $\hat{\beta}_0$, the mathematical formula for which is as follows:

$$S^2_{\hat{\beta}_0} = V(\hat{\beta}_0) = \hat{\sigma}^2\left(\frac{1}{n} + \frac{\bar{X}^2}{S_{XX}}\right)$$

$$S^2_{\hat{\beta}_0} = 77.9\left(\frac{1}{10} + \frac{(49.1)^2}{2048.9}\right)$$

$$S^2_{\hat{\beta}_0} = 99.45$$

$$S_{\hat{\beta}_0} = 9.97$$

Thus, the value of the statistical test t is as follows:

$$t = \frac{\hat{\beta}_0}{S_{\hat{\beta}_0}} = \frac{85.043}{9.97} = 8.53$$

Decision: Since the calculated t value (8.53) is greater than the table value (2.31), that is:

$$|t_{cal.}| = 8.53 > t(0.025, 8) = 2.31$$

So, we reject the null hypothesis and accept the alternative hypothesis, that is, $H_1: \beta_0 \neq 0$.

As for the hypothesis that β_0 a certain value is equal β_{00}, for example, the hypothesis will be as follows

$$H_0: \beta_0 = \beta_{00}$$
$$H_1: \beta_0 \neq \beta_{00}$$

The statistical test t will be shown in the following formula:

$$t = \frac{\hat{\beta}_0 - \beta_{00}}{S_{\hat{\beta}_0}}$$

If $|t_{cal.}| > t(\frac{\alpha}{2}, n-2)$ rejects the null hypothesis H_0 and accepts the alternative hypothesis H_1.